

## Homework Assignment 2 (Due Date: Feb 4, 2014)

1. Consider  $u(x, t)$  which satisfies

$$u_t + uu_x = 0, \quad -\infty < x < +\infty, t > 0$$

with

$$u(x, 0) = \begin{cases} 0, & x \leq 0, \\ x, & 0 \leq x \leq 1, \\ 2 - x, & 1 \leq x \leq 2, \\ 0, & x > 2 \end{cases}$$

Find the solution in different regions of the  $x, t$  plane up until the time that a shock first forms.

2. Consider the following traffic flow problem

$$\rho_t + [Q(\rho)]_x = 0, \quad -\infty < x < +\infty, t > 0$$

where

$$Q(\rho) = U_{max}\rho\left(1 - \frac{\rho}{\rho_j}\right)$$

(a). Let  $\rho(x, 0) = 2 - x$ . Find  $\rho(x, t)$ .

(b). Solve the problem with

$$\rho(x, 0) = \begin{cases} \frac{2\rho_j}{3}, & x < 0, \\ \frac{\rho_j}{6}, & 0 < x < 1, \\ \frac{\rho_j}{3}, & x > 1 \end{cases}$$

Find out the first time when the expansion fan hits the shock. Find the shock curve afterwards.

(c) Solve the problem with

$$\rho(x, 0) = \frac{\rho_j}{8}, \quad -\infty < x < +\infty, \rho(0-, t) = \rho_j, \rho(0+, t) = \frac{\rho_j}{4}$$