

MATH400-201 Homework Assignment 5 (Due Date: March 18, by 6pm, 2016)

1. Solve the following diffusion equation

$$\begin{cases} u_t = ku_{xx}, 0 < x < l \\ u(x, 0) = x, \\ u_x(0, t) = 0, u_x(l, t) = 0 \end{cases} \quad (1)$$

Find out the limit of $u(x, t)$ as $t \rightarrow +\infty$.

2. (a) Find the eigenvalues and eigenfunctions of

$$X'' + \lambda X = 0, 0 < x < l, X(0) = 0, X'(l) = 0$$

(b) Solve the following wave equation

$$\begin{cases} u_{tt} = c^2 u_{xx}, 0 < x < l \\ u(x, 0) = 0, u_t(x, 0) = 2 \sin\left(\frac{3\pi}{2l}x\right) \\ u(0, t) = 0, u_x(l, t) = 0 \end{cases} \quad (2)$$

3. (a) Solve

$$\begin{cases} u_t - u_{xx} = 0, 0 < x < 1, \\ u(x, 0) = \phi(x), 0 < x < 1 \\ u_x(0, t) + 2u(0, t) = 0, u_x(1, t) = 0 \end{cases} \quad (3)$$

by separation of variables. (b) Under what conditions on $\phi(x)$, does the solution to (3) remain bounded as $t \rightarrow +\infty$?

4. For the following eigenvalue problems, find out : (1) how many negative eigenvalues there are (2) the algebraic equations for all positive, zero and negative eigenvalues

$$(a) \quad X'' + \lambda X = 0, 0 < x < 1, \quad 2X'(0) + X(0) = 0, \quad X'(1) + X(1) = 0$$

$$(b) \quad X'' + \lambda X = 0, 0 < x < 1, \quad X'(0) + 2X(0) = 0, \quad X'(1) - 2X(1) = 0$$

5. For the following eigenvalue problems, transform it into standard Sturm-Liouville eigenvalue problem as

$$(p(x)X')' - q(x)X(x) + \lambda w(x)X(x) = 0$$

$$(a) \quad X'' + xX' + \lambda X = 0$$

$$(b) \quad X'' + \frac{1}{x}X' + \lambda X = 0$$

$$(c) \quad X'' + \frac{1}{x}X' - xX' + \lambda X = 0$$