Homework Assignment 3 (Due Date: Feb 12, 2016)

1. (20pts) (a) State the well-posedness criteria for the following backward diffusion equation

$$\begin{cases} u_t + u_{xx} = 0, t > 0, -\infty < x < +\infty, k > 0 \\ u(x,0) = \phi(x) \end{cases}$$

(b) Use the function $u(x,t) = \frac{1}{n} e^{n^2 t} \sin(nx)$ to show that the above problem is ill-posed.

2. (20pts) Use the change of variable to transform the following equations into one of the three standard form

(a) $u_{xx} + 2u_{xy} - u_{yy} = 0$, (b) $u_{xx} - 6u_{xy} + 9u_{yy} - 2u_x = 0$, (c) $4u_{xx} - 4u_{xy} + 5u_{yy} = 0$

3.(20pts) Solve the following wave equation:

$$u_{tt} - u_{xx} = 0$$

$$u(x,0) = x, u_t(x,0) = e^x$$

4. (20pts) Find the solution for

$$\begin{cases} u_{tt} - u_{tx} - 2u_{xx} = 0, t > 0, -\infty < x < +\infty, \\ u(x,0) = \sin(x), u_t(x,0) = 0, -\infty < x < +\infty \end{cases}$$

5. (20pts) Solve the following wave equation

$$u_{tt} = c^2 u_{xx} + \cos(x)$$

$$u(x,0) = \sin(x), u_t(x,0) = 1 + x$$

You may use the d'Alembert's formula for

$$u_{tt} = c^2 u_{xx} + f(x,t)$$

$$u(x,0) = \phi(x), u_t(x,0) = \psi(x)$$

$$u(x,t) = \frac{\phi(x+ct) + \phi(x-ct)}{2} + \frac{1}{2c}\int_{x-ct}^{x+ct} \psi(s)ds + \frac{1}{2c}\int_0^t \int_{x-c(t-s)}^{x+c(t-s)} f(y,s)dyds$$