

## Homework Assignment 2 (Due Date: Feb 2, 2016)

- 1.(10pts) Find the solutions to the following quasilinear problem

$$u_t + (1 - u)u_x = 0, t > 0$$

$$u(x, 0) = 1 - x$$

2. Consider  $u(x, t)$  which satisfies

$$u_t + u^2 u_x = 0, -\infty < x < +\infty, t > 0$$

with

$$u(x, 0) = \begin{cases} 0, & x < 0, \\ 1, & 0 < x < 1, \\ 0, & 1 < x \end{cases}$$

(20pts) Find the solution in different regions of the  $x, t$  plane up until the time that the expansion fan hits the characteristic curve. (10pts) Find the shock curve afterwards.

3. (30pts) Consider the following traffic flow problem

$$\rho_t + [Q(\rho)]_x = 0, -\infty < x < +\infty, t > 0$$

where

$$Q(\rho) = \rho\left(1 - \frac{\rho}{3}\right)$$

Solve the problem with

$$\rho(x, 0) = \frac{3}{8}, -\infty < x < +\infty$$

$$\rho(0-, t) = 3, \rho(0+, t) = \frac{3}{4}$$

4. (30pts) Solve the following fully nonlinear PDE:

$$u_y = \frac{1}{2}u_x^2$$

$$u(x, 0) = 2x$$