MATH400-201 Homework Assignment 1 (Due Date: Jan. 19, 2016)

1. (10points) Solve the following first order PDE and find where the solution is defined in the $x - y$ plane.
$$2u_x + u_y = 0, u(x, x) = e^x$$

2. (10points) Solve the following first order PDE and find where the solution is defined in the $x - y$ plane.
$$u_x + 3x^2yu_y = 0, u(0, y) = y$$

3. (20points) Solve the following first order PDE and find where the solution is defined in the $x - y$ plane.
$$xu_x + (x + y)u_y = u, u(1, y) = y^2, 0 \le y \le 1$$

4. (20points) Solve $u_t + (x + 1)u_x = 4u$ for $x > 0, t > 0$ with $u(0, t) = t$ and $u(x, 0) = 1$.

5. (20points) Solve the following first order PDE and find where the solution becomes unbounded in the $x - y$ plane.

$$u_x + e^x u_y = -u^2, \ u = 1 \ \text{ on the curve } y = 2e^x$$

6. (20points) Let $u(x, y)$ solve the first order PDE

$$xu_x + yu_y = x^2u$$

(a). Find the general solutions. (b) Suppose we put $u = h(x)$ on $y = x$. Derive the condition that $h(x)$ must satisfy for a solution to exist.