MATH305-201, HW#8, 2016-2016, Due: 5:30pm, March 13

1. Let f be an entire function such that

$$Re(f)+Im(f) = u+v > 0$$

Show that f is a constant. Hint: consider e^{cf} for some c.

2. Show that $\max_{|z|\le 1} |az^n+b| = |a|+|b|$

3. Find (a) $\max_{|z|\le 1} |(z-1)(z+1)/2|$ (b) $\max_{|z|\le 1} |e^{z+z^2}|$

4. Prove that the polynomial $\frac{1}{3}z^{10}+\frac{1}{4}z^6+\frac{1}{6}z^3+\frac{1}{5}z+1=0$ has no roots z inside $|z|\le 1$

Hint: Lecture Not 5, Example 4 on page 23.

5. Use Nyquist criterion to find the number of zeroes of

$f(z) = z^3+2z^2+4$ in right half-plane $= \{Re(z)>0\}$

6. Find the number of zeroes of

$f(z) = z^3+2z^2+4z+2$ in the right half-plane

Hint: Use Nyquist criterion

7. Find the number of zeroes of

$f(z) = z^3+z^2+4z+1$ in the right half-plane.

As a consequence, show that the solutions to the following ODE

$y'''+y''+4y'+y=0$, $y(0)=1$, $y'(0)=-1$, $y''(0)=2$

approach to zero as $t\to+\infty$.