MATH305-201-2016/2017 Homework Assignment 2 (Due Date: Jan. 23, 2017, by 5:30pm, in class or at my office)

1. Find all values of the following
   (a)$(i-1)^{1/3}$; (b) $(\frac{2i}{1-\sqrt{3}i})^{1/6}$

2. Let m and n be positive integers that have no common factor. Prove that the set of numbers $(z^{1/n})^m$ is the same as the set of numbers $(z^m)^{1/n}$. Use this result to find all values of $(1-i)^{3/2}$.

3. Write the following functions in the form $w = u(x,y) + iv(x,y)$.
   (a) $f(z) = \frac{z+i}{z+1}$; (b) $f(z) = \frac{e^z}{z}$

4. Describe the image of the following sets under the following maps
   (a) $f(z) = iz + 5$ for $S = \{Re(z) > 0\}$; (b) $f(z) = \frac{z-1}{z+1}$ for $S = \{|z| < 3\}$; (c) $f(z) = -2z^3$ for $S = \{|z| < 1, 0 < Arg z < \frac{\pi}{2}\}$

5. Describe the image of the following sets under the map $w = e^z$
   (a) $S = \{Re(z) = 1\}$; (b) $S = \{Im(z) = \frac{\pi}{4}$; (c) $S = \{0 \leq Im(z) \leq \frac{\pi}{4}\}$

6. The Joukowski map is defined by

$$w = f(z) = \frac{1}{2}(z + \frac{1}{z})$$

   Show that J maps the circle $S = \{|z| = r\}$ $(r > 0, r \neq 1)$ onto an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

7. Show that the function $w = z^2$ maps the line $S = \{Re(z) = 1\}$ into a parabola.

8. Prove that $|e^{-z^3}| \leq 1$ for all z with $-\frac{\pi}{6} \leq Arg(z) \leq \frac{\pi}{6}$.

9. Show that the function $f(z) = \bar{z}$ is continuous everywhere but not differentiable anywhere.

10. Let

$$f(z) = \begin{cases} (x^{4/3}y^{5/3} + ix^{5/3}y^{4/3})/(x^2+y^2), \text{if } z \neq 0; \\ 0 \text{ if } z = 0 \end{cases}$$

Show that the Cauchy-Riemann equations hold at $z = 0$ but f is not differentiable at $z = 0$.

$*$: Only five problems will be chosen to be marked but I suggest that you do all of the them.