

Math 257/316 Assignment 2

Due Friday Jan 23 in class

Problem 1. Find the first six non-zero terms in the power series $y = \sum_{n=0}^{\infty} a_n(x - x_0)^n$ of the general solution of the following second-order, linear, homogeneous ODEs, centred at the indicated point x_0 :

a) $y'' + x^2y' + y = 0, \quad x_0 = 0$

b) $y'' - xy' + y = 0, \quad x_0 = 3,$

writing the solutions in terms of a_0 and a_1 .

Problem 2. Determine a power series solution centred about $x_0 = 0$ for the following initial value problem

$$(x^2 - 4x + 4)y'' + (4x - 8)y' + 2y = 0, \quad y(0) = 1/2, \quad y'(0) = 1/4.$$

Can you express your answer as a familiar function?

Problem 3. For each of the following equations find all singular points, and determine whether each one is regular or irregular:

a) $(x^2 - 1)y'' + y = 0$

b) $x(x + 1)^2y'' + y' = 0$

c) $x^2y'' + y' + \frac{1}{\sin(x)}y = 0$

Problem 4. Verify that the following equation has a regular singular point at $x = 0$, and find the first three non-zero terms in each of two linearly independent series solutions (valid for $x > 0$): $2x^2y'' + 5xy' + (x + 1)y = 0$