Chapter 3.

1. Solve $y''+2y'+2y=0$, $y(0)=2$, $y'(0)=1$

2. Solve $y''+2y'+2y=\cos t$, $y(0)=0$, $y'(0)=0$

3. Solve $y''+2y'+2y=e^{-t}\sin t$, $y(0)=0$, $y'(0)=0$

4. Solve $y''+2y'+y=e^{-t}$, $y(0)=0$, $y'(0)=0$

5. Solve $y''+3y'+2y=\cos t+e^{-t}$, $y(0)=0$, $y'(0)=0$

6. Compute the Wrongskian of

$$2t^2y''+t\ln t\, y'-\sin t\, y=0, \quad t>0$$

7. Let $y_1=e^t$ be a solution of

$$(1-t)y''+ty'-y=0, \quad 0<t<1$$

Use the reduction of order to find y_2

8. Solve $y''+9y=9\sec^2(3t)$, $0<t<\frac{\pi}{6}$

9. ~~Find the~~ Solve $t^2y''-2ty'+2y=0$

10. Let $y_1=1+t$ be a solution of

$$ty''-(1+t)y'+y=0, \quad t>0$$

Find y_2. Compute the Wronskian. Solve

$$ty''-(1+t)y'+y=t^2e^{2t}$$

Practice Problems

Chapter 2

1. Solve $y' + \frac{2}{t}y = \frac{\cos t}{t^2}$, $y(\pi) = 0$, $t > 0$

2. Solve $\frac{dy}{dx} = -\frac{x^2+xy+y^2}{x^2}$, $y(1) = 1$

3. Solve $\frac{dy}{dx} = \frac{4y-3x}{2x-y}$, $y(1) = 1$

4. Solve $t^2 y' + 2ty = y^3$, $t > 0$

5. Find the critical points for the following population models and classify the stability / instability of the critical points

(a) $\frac{dy}{dt} = y(y-1)(y-3)$, $y \geq 0$

(b) $\frac{dy}{dt} = y \cos y$, $y \geq 0$

(c) $\frac{dy}{dt} = (e^y - 1)(y-2)$, $y \geq 0$

(d) $\frac{dy}{dt} = (2-y)\ln(y+1)$, $y \geq 0$

6. Solve $y' = \frac{3x^2 - e^x}{2y-5}$, $y(0) = 1$ and state the Interval of Existence

Chapter 7

Consider
$$X' = \begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix} X + \begin{pmatrix} 3t-2 \\ e^{-t} \end{pmatrix} \quad -(1)$$

1. Use method of diagonalization to solve (1)
2. Use method of undetermined coefficients to solve (1)
3. Use method of variation of parameters to solve (1)
4. Find $\Phi(t)$ of $X' = \begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix} X$ such that

   $\Phi(0) = I$

5. Find the general solution of
$$x' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} x + \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}.$$

6. Find the general solution of
$$x' = \begin{pmatrix} 4 & -2 \\ 8 & -4 \end{pmatrix} + \begin{pmatrix} t^{-3} \\ -t^{-2} \end{pmatrix}.$$

7. Solve $x' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} x + \begin{pmatrix} \csc t \\ \sec t \end{pmatrix}, \ \frac{\pi}{2} < t < \pi$

8. Compute Wronskian of $X' = \begin{pmatrix} t & e^{t^2} \\ \frac{1}{t^2+1} & \ln t \end{pmatrix} X, \ t > 0.$

Chapter 6

1. Use method of Laplace Transform to solve

(a) $y'' - y' - 6y = 0, \quad y(0) = 1, \quad y'(0) = -1$

(b) $y'' + 3y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = 0.$

(c) $y'' + 4y = \begin{cases} 1, & 0 < t < \pi \\ 0, & \pi < t < \infty \end{cases}, \quad y(0) = 1, \quad y'(0) = 0$

(d) $y'' + y = \begin{cases} t, & 0 \le t < 1 \\ 2 - t, & 1 \le t < 2 \\ 0, & 2 \le t < \infty \end{cases}, \quad y(0) = 0, \quad y'(0) = 0$

2. Use method of Laplace Transform to solve

(a) $y'' + 3y' + 2y = \cos t, \quad y(0) = 0, \quad y'(0) = 1$

(b) $y'' + 2y' + 2y = 3u_1(t), \quad y(0) = 1, \quad y'(0) = 1$

(c) $y'' + 4y' + 5y = u_2(t) + (\sin t)\,\delta(t-3), \quad y(0) = 0, \quad y'(0) = 0$

(d) $y'' + 2y' + 5y = e^{t} + e^{2t}\delta(t-4) - 10u_5(t), \quad y(0) = 0, \quad y'(0) = 0$

Chapter 10

1. Find the Fourier series for the given function

(a) $f(x) = \begin{cases} x, & -\pi \le x < 0 \\ 0, & 0 \le x < \pi \end{cases}$, $f(x+2\pi) = f(x)$

(b) $f(x) = \begin{cases} 0, & -2 \le x \le -1 \\ x & -1 < x < 1 \\ 0, & 1 \le x < 2 \end{cases}$; $f(x+4) = f(x)$

(c) $f(x) = \begin{cases} x+2, & -2 \le x < 0 \\ 2-2x, & 0 \le x < 2 \end{cases}$; $f(x+4) = f(x)$

2. Find the values of the Fourier series at values $x = -\frac{\pi}{2}$, $x = 0$, $x = \frac{\pi}{2}$ of 1 (a)

3. Use method of separation of variables to solve

$$\begin{cases} u_t = u_{xx}, & 0 < x < 2\pi \\ u(x,0) = \sin x - \sin 4x \\ u(0,t) = 0, \ u(2\pi, t) = 0 \end{cases}$$

4. Solve

$$\begin{cases} u_{tt} = u_{xx}, & 0 < x < \pi \\ u(x,0) = \cos 2x, \ u_t(x,0) = \cos^2 x \\ u_x(0,t) = 0, \ u_x(\pi, t) = 0 \end{cases}$$

5. Solve

$$\begin{cases} u_t = u_{xx}, & 0 < x < \pi \\ u(x,0) = \sin^2 x \\ u_x(0,t) = 0, \ u(\pi, t) = 0 \end{cases}$$