MATH400-101-2020 Homework Assignment 2 (Due Date: September 27, 2019, by 6pm)

Homework is admitted until 6pm on September 27, 2019. Graded homework is placed in a cardboard box outside my office for one week for you to pick up. Afterwards unclaimed homework is moved to a drawer of a file cabinet near my office. Your assignments are organized in the alphabetic order of last names. For other people's convenience, please do not change this order when you pick up your assignment.

10points each

1. Find the general solutions to the following first order PDE: $u_x + yu_y = 2u + y$.
2. Find the solutions to the following quasilinear problem

$$(a)\ u_t+uu_x = 0, t > 0, u(x,0) = x-3; \quad (b)\ u_t+(1-u)u_x = 0, t > 0, u(x,0) = 1-2x; \quad (c)\ u_t+u^2u_x = 0, t > 0, u(x,0) = x$$

3. Find out the maximum breaking up $t = t_B$ for the following quasilinear problem

$$(a)\ \ u_t + u^2u_x = 0,\ u(x,0) = \frac{1}{\sqrt{1+x^2}} : \quad (b)u_t + u^3u_x = 0, u(x,0) = e^{-x^2}$$

4. Find the expansion fan of the form $u = U(\frac{x}{t})$ for the following quasilinear problem

$$(a)\ \ 2u_t + u^2u_x = 0; \quad (b)u_t + (2u - u^2)u_x = 0$$

5. Use expansion fan to solve the following quasilinear first order problem

$$u_t + u^3u_x = 0;$$

$$u(x,0) = \begin{cases} -1, -\infty < x < 1; \\ 1, x > 1 \end{cases}$$

6. Solve the following quasilinear first order problem for $\mathbf{t < 1}$

$$u_t + uu_x = 0;$$

$$u(x,0) = \begin{cases} 0, -\infty < x < 0; \\ 1, 0 < x < 1; \\ -1, x > 1 \end{cases}$$

7. Consider $u(x,t)$ which satisfies

$$u_t + (u-1)u_x = 0,\ -\infty < x < +\infty, t > 0$$

with

$$u(x,0) = \begin{cases} 0,\ x < 0, \\ 2,\ 0 < x < 2, \\ 1,\ 2 < x \end{cases}$$

(a) Find the solution in different regions of the x, t plane up until the time that **the shock curve hits the expansion fan**. (b) Find the shock curve afterwards.

8. Consider the following traffic flow problem

$$u_t + (2-u)u_x = 0, -\infty < x < +\infty,\ t > 0$$

Solve the problem with

$$u(x,0) = \frac{1}{2}, -\infty < x < +\infty$$
$$u(0-,t) = 5,\ u(0+,t) = 1, t > 0$$

9. Solve the following fully nonlinear PDE:

$$u_y = u_x^2, \quad u(x,0) = x$$

10. Solve the following fully nonlinear PDE:

$$u_x u_y = u,\ u(x,0) = x+1$$