Practice Problems

The following problems are for practice only.

First Order

1. Solve $ty' + 2y = \frac{(\sin t)}{t^2}$, $y\left(\frac{\pi}{2}\right) = 1$

   and state the interval of existence

2. Solve $\begin{cases} (\sin t)\,y' \end{cases}$ [illegible] $2(\cos t)\,y = \sin^3 t$

   $y\left(\frac{\pi}{2}\right) = 1$

   and state the interval of existence

3. Solve ty' [illegible] $y = \frac{\sin t}{t^2}\, y^3$

   $y\left(\frac{\pi}{2}\right) = 1$

   and state the interval of existence

4. Solve $(\sin t)\,y'$ [illegible] $\cos t\; y = (\sin^3 t)\, y^2$

   $y\left(\frac{\pi}{2}\right) = 1$

   and state the interval of existence

5. Solve $y' = \frac{2x}{1+2y}$, $y(2) = -1$ and state the interval of existence

6. Solve $y' = \frac{3x^2 - e^x}{3y-5}$, $y(0)=1$ and state the interval of existence

7. Solve $\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$

8. Solve $\frac{dy}{dx} = 1 + \frac{3xy+y^2}{x^2}$

9. Solve $\frac{dy}{dx} = ry \ln(K/y)$

subject to the initial condition $y(0) = y_0$

10. Locate the critical points and state the stability/instability of the critical points for

$$\frac{dy}{dx} = \sin y$$

Second order

11. State the definition of fundamental solutions

12. Consider $t^2 y'' + (1+t) y' + \sin t \, y = 0$.
Let Y_1, Y_2 be two solutions.

Suppose $W[Y_1, Y_2](1) = 3$.

Find $W[Y_1, Y_2](5)$

13. Find the Wronskian of $x^2 y'' + x y' + (x^2 - 1) y = 0$.

14. Solve the following equations

(a) $y'' + 3y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = 0$

(b) $y'' + 3y' + 4y = 0, \quad y(0) = 0, \quad y'(0) = 1$

(c) $y'' + 4y' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 1$

(d) $2y'' + 3y' + 6y = 0, \quad y(0) = 0, \quad y'(0) = 1$

15. Find the general solutions of the following equations

(a) $y'' + 3y' + 2y = e^{t}\cos t$

(b) $y'' + 3y' + 2y = te^{-t}$

(c) $y'' + 9y = te^{3t} + 3\sin(3t)$

(d) $y'' + 4y' + 4y = t^2 e^{2t} - \sin t$

(e) $y'' + 2y' + 4y = e^{t}\sin\sqrt{3}\,t$

16. Find the general solutions of

$\cancel{y'' + 2y' + 5y}$

(a) $y'' + 4y' + 4y = t^{-2}e^{-2t}, \quad t > 0$

(b) $y'' + 4y = \sec^2 2t, \quad 0 < t < \frac{\pi}{4}$

(c) $t^2 y'' - 2y = 3t^2 - 1, \quad t > 0; \quad y_1 = t^2, \quad y_2 = t^{-1}$

(d) $4y'' + y = 2\sec(t/2), \quad -\pi < t < \pi.$

17. Find a general formula for
$y'' + y = g(t), \quad y(t_0) = 0, \quad y'(t_0) = 0.$

18. Find the general solutions of

(a) $2t^2y''+3ty'+y=0$

(b) $t^2y''+2ty'+y=0$

(c) $t^2y''-3ty'-2y=0$

(d) $t^2y''+ty'+(t^2-\frac{1}{4})y=0$

$y_1=t^{-\frac{1}{2}}\sin t$

(e) $(1-t)y''+ty'-y=0$

$y_1=e^t$

(f) $ty''-y'+4t^3y=0$, $y_1=\sin t^2$

(g) $t^2y''-t(t+2)y'+(t+2)y=0$

$y_1=t$

18. In the following, the first solution of the homogeneous problem is given. Solve the problem

(a) $ty'' - (1+t)y' + y = t^2 e^{2t}, \quad t > 0$

$y_1(t) = 1 + t$

(b) $(1-t)y'' + ty' - y = 2(t-1)^2 e^{-t}, \quad 0 < t < 1,$

$y_1 = e^t$

(c) $t^2 y'' - 3ty' + 4y = t^2 \ln t, \quad t > 0$

$y_1 = t^2$

(d) $t^2 y'' + ty' + (t^2 - \frac{1}{4})y = 3t^{3/2} \sin t, \quad t > 0$

$y_1 = t^{-\frac{1}{2}} \sin t$