MATH256-103 Homework Assignment 2 (Due Date: by 5pm on October 2nd 2018)

Hand in your homework either in class, or by 5pm at my office, on October 2nd. Graded homework is placed in a cardboard box outside my office for one week for you to pick up. Afterwards unclaimed homework is moved to a drawer of a file cabinet near my office. Your assignments are organized in the alphabetic order of last names. For other people's convenience, please do not change this order when you pick up your assignment.

1. Use Abel's formula to find the Wronskian for the following second order ODEs:

$$(a)y'' + xy' + 2y = 0,\ (b)\ x^2y'' - 2xy' + (x^2+1)y = 0,\ (c)\sin t\, y'' + 2\cos t y' + t^4 y = 0,\ (d)(1+x^2)y'' + xy' + x^5 y = 0$$

2. Find the general solutions to the following second order ODEs:

$$(a)y'' + 3y' - 4y = 0, (b)2y'' + 3y' + y = 0, (c)y'' + 2y' + 5y = 0, (d)y'' + 9y = 0,\ (e)y'' + 6y' + 9y = 0$$

3. Solve

$$(a)y'' + 2y' + 3y = 0, y(0) = -1, y'(0) = 2; (b)y'' + 4y' + 3y = 0, y(-1) = 2, y'(-1) = 1;$$
$$(c)9y'' - 12y' + 4y = 0, y(0) = 2, y'(0) = -1; (d)2y'' - 5y' = 0, y(1) = 1, y'(1) = -1.$$

4. Use the method of reduction of order to find a second solution

$$(a)t^2y'' + 3ty' + y = 0, t > 0, y_1 = t^{-1}; (b)t^2y'' - t(t+2)y' + (t+2)y = 0, t > 0, y_1 = t;$$
$$(c)(x-1)y'' - xy' + y = 0, x > 1, y_1 = e^x; (d)x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0, x > 0, y_1 = x^{-1/2}\sin x$$

You may use the following formula: $y_2 = y_1 \int \frac{W}{y_1^2} dt$

5. Find the general solutions of the following Euler type equations

$$(a)t^2y'' + 3ty' - 2y = 0,\ (b)2t^2y'' - 4ty' + y = 0,\ (c)t^2y'' + 3ty' + y = 0,\ (d)t^2y'' + 5ty' + 13y = 0$$

6. Use the method of undetermined coefficients to find the general solutions of the following second order ODE

$$(a)y'' + 2y' + 2y = \sin t, (b)\ y'' + 2y' + 2y = 3t^2,\ (c)\ y'' + 2y' + 2y = e^{-t}\sin(2t),$$
$$(d)\ y'' + 2y' + 2y = e^{-t}\cos(t),\ (e)y'' + 2y' + y = 2\cosh(t)$$

7. Write the general form of the special solutions to

$$(a)y'' + y = -t^3 + t^2e^t + 2t\cos t,\ (b)y'' + 4y' + 3y = \sinh t + t^2 - 2 + \cos(2t)$$

8. If a, b, c are all positive constants, show that all solutions to $ay'' + by' + cy = 0$ approach zero as $t \to +\infty$.

$$\text{Notations} : \cosh t = \frac{e^t + e^{-t}}{2},\ \sinh t = \frac{e^t - e^{-t}}{2}$$