Chapter 6

1 (a)

$$s^2 Y(s) - s\,y(0) - y'(0) - (s Y(s) - y(0)) - 6Y = 0$$

$$(s^2 - s - 6) Y(s) = s\,y(0) + y'(0) - y(0) = s - 1 + 1 = s$$

$$Y(s) = \frac{s}{s^2 - s - 6} = \frac{s}{(s-3)(s+2)} = \frac{A}{(s-3)} + \frac{B}{s+2}$$

$$A(s+2) + B(s-3) = s \Rightarrow A + B = 1, \quad 2A - 3B = 0$$

$$B = -2, \quad A = 3$$

$$Y(s) = \frac{3}{s-3} - \frac{2}{s+2}$$

$$y(t) = 3e^{3t} - 2e^{-2t}$$

(b) $s^2 Y(s) - s + 3(sY(s) - 1) + 2Y(s) = 0$

$$Y(s) = \frac{s+3}{s^2 + 3s + 2} = \frac{s+3}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$s + 3 = A(s+2) + B(s+1)$$

$$A + B = 1, \quad 2A + B = 3 \Rightarrow A = +2, \quad B = 1$$

$$Y(s) = \frac{2}{s+1} - \frac{1}{s+2}$$

$$y(t) = 2e^{-t} - e^{-2t}$$

(c) $g(t) = 1 \cdot (u_0 - u_\pi) = 1 - u_\pi$

$$G(s) = \frac{1}{s} - \frac{e^{-\pi s}}{s}$$

$$s^2 Y(s) - s + 4Y(s) = \frac{1}{s} - \frac{e^{-\pi s}}{s}$$

$$Y(s) = \frac{s}{s^2+4} + \frac{1}{s(s^2+4)} - \frac{e^{-\pi s}}{s(s^2+4)}$$

$$\frac{1}{s(s^2+4)} = \frac{A}{s} + \frac{Bs}{s^2+4} + \frac{C}{s^2+4}$$

$$\Rightarrow \quad A = \frac{1}{4}, \quad B = -\frac{1}{4}, \quad C = 0$$

$$\frac{1}{s(s^2+4)} = \frac{\frac{1}{4}}{s} - \frac{\frac{1}{4}s}{s^2+4}$$

$$y(t) = \cos 2t + \frac{1}{4} - \frac{1}{4}\cos 2t - u_\pi(t)\left\{\frac{1}{4} - \frac{1}{4}\cos 2(t-\pi)\right\}$$

(d) $g(t) = t(1-u_1) + (2-t)(u_1-u_2)$

$$= t - tu_1 + (2-t)u_1 + (t-2)u_2$$

$$= t + 2(1-t)u_1 + (t-2)u_2$$

$$G(s) = \frac{1}{s} - \frac{2}{s^2}e^{-s} + \frac{e^{-2s}}{s^2}$$

$$Y(s) = \frac{1}{s(s^2+1)} - \frac{2e^{-s}}{s^2(s^2+1)} + \frac{e^{-2s}}{s^2(s^2+1)}$$

$$\frac{1}{s(s^2+1)} = \frac{1}{s} - \frac{s}{s^2+1}, \quad \frac{1}{s^2(s^2+1)} = \frac{1}{s^2} - \frac{1}{s^2+1}$$

$$y(t) = 1 - \cos t - 2u_1(t)\big(t-1-\sin(t-1)\big)$$
$$+ u_2(t)\big(t-2-\sin(t-2)\big)$$

2. (a) $s^2Y(s) - s\cdot 0 - 1 + 3(sY(s) - 0) + 2Y(s) = \frac{s}{s^2+1}$

$$(s^2+3s+2)Y(s) = 1 + \frac{s}{s^2+1}$$

$$Y(s) = \frac{1}{(s^2+3s+2)} + \frac{s}{(s^2+1)(s^2+3s+2)}$$

$$\frac{1}{s^2+3s+2} = \frac{1}{s+1} - \frac{1}{s+2}$$

$$\frac{s}{(s^2+1)(s^2+3s+2)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{Cs}{s^2+1} + \frac{D}{s^2+1}$$

$$s = A(s+2)(s^2+1) + B(s^2+1)(s+1) + Cs(s+1)(s+2) + D(s+1)(s+2)$$

$s=-2 \Rightarrow \quad B = \frac{2}{5}$

$s=-1 \Rightarrow \quad A = -\frac{1}{2}$

$A+B+C=0 \quad \Rightarrow \quad C = \frac{1}{2} - \frac{2}{5} = \frac{1}{10}$

$2A+B+2D=0 \quad \Rightarrow \quad D = \frac{3}{10}$

$$Y(s) = \frac{1}{s+1} - \frac{1}{s+2} + \frac{-\frac{1}{2}}{s+1} + \frac{\frac{2}{5}}{s+2} + \frac{\frac{1}{10}s}{s^2+1} + \frac{\frac{3}{10}}{s^2+1}$$

$$y(t) = \frac{1}{2}e^{-t} - \frac{3}{5}e^{-2t} + \frac{1}{10}\cos t + \frac{3}{10}\sin t$$

(b). $s^2Y(s) - s - 1 + 2(sY(s) - 1) + 2Y(s) = 3\frac{e^{-s}}{s}$

$$(s^2+2s+2)Y(s) = s+3+3\frac{e^{-s}}{s}$$

$$Y(s) = \frac{s+3}{s^2+2s+2} + \frac{3e^{-s}}{s(s^2+2s+2)}$$

$$\frac{s+3}{s^2+2s+2} = \frac{s+1}{(s+1)^2+1} + \frac{2}{(s+1)^2+1}$$

$$\frac{1}{s(s^2+2s+2)} = \frac{A}{s} + \frac{B(s+1)}{(s+1)^2+1} + \frac{C}{(s+1)^2+1}$$

$$1 = A\left((s+1)^2+1\right) + Bs(s+1) + Cs$$

$$A=1, \quad B=-1, \quad C=0$$

$$Y(s) = \frac{s+1}{(s+1)^2+1} + \frac{2}{(s+1)^2+1} + \frac{1}{s} - \frac{(s+1)}{(s+1)^2+1}$$

$$= \frac{2}{(s+1)^2+1} + \frac{1}{s}$$

$$y(t) = 1 + 2e^{-t}\sin t$$

(c) $$(s^2+4s+5)Y(s) = \frac{e^{-2s}}{s} + (\sin 3)\, e^{-3s}$$

$$Y(s) = \frac{e^{-2s}}{s(s^2+4s+5)} + \frac{\sin 3}{s^2+4s+5}\, e^{-3s}$$

$$\frac{1}{s(s^2+4s+5)} = \frac{A}{s} + \frac{B(s+2)}{(s+2)^2+1} + \frac{C}{(s+2)^2+1}$$

$$1 = A\left((s+2)^2+1\right) + Bs(s+2) + Cs$$

$$A=1 \quad B=-1, \quad C=0$$

$$Y(s) = \left(\frac{1}{s} - \frac{s+2}{(s+2)^2+1}\right)e^{-2s} + \frac{\sin 3}{(s+2)^2+1}\, e^{-3s}$$

$$y(t) = u_2(t)\left(1 - e^{-2(t-2)}\cos(t-2)\right) + (\sin 3)\, u_3(t)\, e^{-2(t-3)} \sin(t-3)$$

(d)

$$(s^2+2s+5)\,Y(s) = \frac{1}{s-1} + e^8 \cdot \frac{e^{-4s}}{1} - 10\,\frac{e^{-5s}}{s}$$

$$Y(s) = \frac{1}{(s-1)((s+1)^2+4)} + \frac{e^8 e^{-4s}}{(s-1)((s+1)^2+4)} - \frac{10\, e^{-5s}}{s((s+1)^2+4)}$$

$$\frac{1}{(s-1)((s+1)^2+4)} = \frac{A}{s-1} + \frac{B(s+1)}{(s+1)^2+4} + \frac{C}{(s+1)^2+4}$$

$$A = \frac{1}{8}, \quad B = -\frac{1}{8} \qquad C = -\frac{1}{4}$$

$$\frac{1}{s((s+1)^2+4)} = \frac{A}{s} + \frac{B(s+1)}{(s+1)^2+4} + \frac{C}{(s+1)^2+4}$$

$$A = \frac{1}{5}, \quad B = -\frac{1}{5} \qquad C = -\frac{1}{5}$$

$$\begin{aligned} y(t) = {} & \frac{1}{8}e^{t} - \frac{1}{8}e^{-t}\cos 2t - \frac{1}{4}e^{-t}\sin 2t \\ & + e^8 u_4(t)\left(\frac{1}{8}e^{t-4} - \frac{1}{8}e^{-(t-4)}\cos 2(t-4) - \frac{1}{4}e^{-(t-4)}\sin 2(t-4)\right) \\ & - 10\,u_5(t)\left(\frac{1}{5} - \frac{1}{5}e^{-(t-5)}\cos 2(t-5) - \frac{1}{5}e^{-(t-5)}\sin 2(t-5)\right) \end{aligned}$$