Review of MATH253-105

Part I (a) operations of vectors: $\vec{u}\cdot\vec{v}$, $\vec{u}\times\vec{v}$

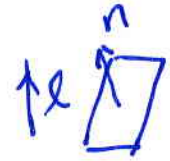
(b) Applications

b1. Distance from point to plane

b2 Equation of plane in R^3, determined by

- three points, P_1, P_2, P_3, $[(P_2-P_1)\times(P_3-P_1)]\cdot(x-P_1)=0$
- One point + two vectors: $(\vec{a}\times\vec{b})\cdot(x-P_1)=0$
- normal to the plane: $\vec{n}\cdot(x-P)=0$

b3 Area of ~~parallelogram~~ parallelogram  $|\vec{a}\cdot(\vec{b}\times\vec{c})|$



b4 Relations between lines and planes:

- A line and a plane either intersect or parallel (do not intersect): $\vec{l}\cdot\vec{n}=0$ (parallel)
- two planes either intersect or parallel

$$\vec{n}_1\cdot\vec{n}_2=0$$

Part II: Two-dimensional ~~quadratic~~ quadratic surfaces & contours

$$z=ax^2+bxy+cy^2+ex+fy+d \qquad z=x^2+y^2+xy,\quad z=x^2+y^2-3xy$$

Contours: $z=x^2+y^2$, $z=\sqrt{x^2+y^2}$, $z=x^2-y^2$,

Part III: Calculus of two-variable functions

- $z=f(x,y)$, continuity at points
- $z=f(x,y)$, f_x, f_y at points, differentiability at points
- Compute $f_x, f_y, f_{xy}, f_{yx}, f_{xxy}, \ldots$
- Implicit functions

Part IV: Applications of Partial derivatives

- gradient, ∇f, ~~steepest~~ steepest descent, directional derivatives
- tangent planes. $z = z_0 + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

tangent planes for implicit functions: $F(x,y,z)=0$

$$F_x(x_0,y_0,z_0)(x-x_0) + F_y(x_0,y_0,z_0)(y-y_0) + F_z(x_0,y_0,z_0)(z-z_0)=0$$

normals $\langle -f_x, -f_y, 1\rangle$, or $\langle F_x, F_y, F_z\rangle$

- total differentials: $dz = f_x\,dx + f_y\,dy$
- Critical points, $\nabla f = 0 \Leftrightarrow f_x=0,\ f_y=0$
- classification of critical points; local contour; $D=\begin{pmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy}\end{pmatrix}$
- Finding local max/min, global max/min
- Method of Lagrange for minimization problems with constraints

Part V: Integrals

- $\iint_D f\,dA$ $<$ type 0, type I, type II domains
- exchange of order $dx\,dy$, $dy\,dx$
- polar $\iint_D f\,dA = \iint_{D'} f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$
- $\iiint_D f\,dV$, exchange of order $dx\,dy\,dz$, $dx\,dz\,dy$, $dy\,dx\,dz$, ...
- $\iiint_D f\,dV = \iiint f(\ldots)\, r\,dz\,dr\,d\theta$, cylindrical
- $\iiint_D f\,dV = \iiint f(\quad)\, \rho^2\sin\phi\, d\rho\, d\phi\, d\theta$, spherical
- Applications $<$ center of mass, volume, area; moment of inertial — surface area $\iint_D \rho\sqrt{1+f_x^2+f_y^2}$

Examples 1: $ax+by+cz=d$, z-axis.

intersect, doesnot intersect (parallel), orthogonal

Ex. 2: Contour

$z=x^2+y^2$, $z^2=x^2+y^2$

$z=x^2+y^2-xy$, $z=x^2+y^2-2xy$, $z=x^2+y^2-3xy$

Ex. 3 $f(x,y)=\begin{cases}\frac{xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & \end{cases}$

$\cancel{f(0,0)}$ ∃, f is continuous at $(0,0)$

Ex. 4. tangent planes: $\cancel{e^{x+y}(2\cos z+1) - \frac{e^{z^2}}{\sqrt{x^2+y^2}} = 0, \ (0,0,0)}$

$\cancel{F_x = e^{y}\cos z + \frac{x}{(x^2+y^2)^{3/2}}}$ $x^2+\frac{y^2}{4}+\frac{z^2}{9}=3$ at $(1,2,3)$

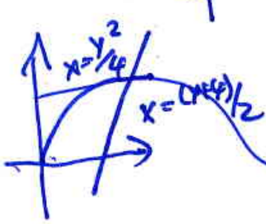
$F_x(x-x_0)+F_y(y-y_0)+F_z(z-z_0)=0$

Ex. 5 Total differentials: Volume of cone $\frac{1}{3}\pi r^2 h$

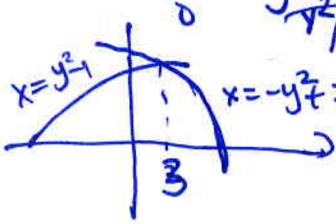
Ex. 6. double ~~of on~~ integrals; exchange of order

$\cancel{\int_0^1\int_{x^2}^{x} e^{-y^2}\,dy\,dx}$ $x^2\le y\le x$

$\cancel{\begin{cases} y\le x\le\sqrt{y}\\ 0\le y\le 1\end{cases}}$



$$\int_0^4\int_{y^2/4}^{(y+4)/2} dx\,dy = \int_0^2\int_0^{2\sqrt{x}} dy\,dx + \int_2^4\int_{2x-4}^{2\sqrt{x}} dy\,dx$$



$$\int_0^2\int_{y^2-1}^{-y^2+7} f\,dx\,dy = \int_{-1}^3\int_0^{\sqrt{x+1}} f\,dy\,dx + \int_3^7\int_0^{\sqrt{7-x}} f\,dy\,dx$$

~~Ex. 7. Triple integrals~~

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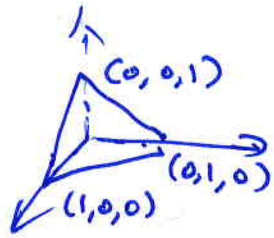
$$\iiint_E f\,dV$$



Ex. 8. Spherical. coordinates

cylindrical

$$\iint \int_0^{\sqrt{x^2+y^2}} \frac{1}{1+x^2+y^2}\,dz\,dx\,dy$$