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Computational design of art-inspired metamaterials

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In recent years, there has been a surge of interest in the design of mechanical metamaterials for different science and engineering applications. In particular, various computational approaches have been developed to facilitate the systematic design of art-inspired metamaterials including origami and kirigami metamaterials. In this Comment, we highlight the recent advances and discuss the outlook for the computational design of art-inspired metamaterials.

Mechanical metamaterials are artificial materials that can exhibit highly unusual physical properties such as negative Poisson's ratios and stunning shape-morphing effects. Inspired by the traditional paper folding and cutting art, there has been an increasing interest in origami-based and kirigami-based metamaterials, which can easily transform from flat sheets into complex three-dimensional structures with programmable mechanical properties. These metamaterials have been widely applied to robotics, soft electronics, biomedical devices, and many other science and engineering problems. For instance, both origami and kirigami have been used for the design of programmable soft robots with the ability to move and function in complex physical environments, and the design of soft grippers with substantial grasping strength. They have also found potential applications in the fabrication of stretchable circuits, 3D nanostructures, and even foldable space telescopes for space exploration.

Over the past several decades, numerous works have studied different well-known origami patterns such as the Miura-ori pattern, Yoshimura pattern, and waterbomb origami pattern. Common kirigami patterns such as the rotating squares pattern and the triangle-based kagome pattern have also been analyzed in terms of their mechanical and geometrical properties. Building upon the standard origami and kirigami patterns, some later works have explored their generalizations and created more complex structures. To further facilitate the design of metamaterials, it is important to develop computational approaches that allow for the creation of a large variety of structures with different desired shape morphing, stiffness, multi-stability and other mechanical properties in an efficient and systematic manner.

Computational origami design

Given a flat sheet of materials, how can we change the shape of it? In origami design, one addresses this question by introducing a crease pattern in the 2D sheet and folding it accordingly into a 3D configuration. For instance, in the well-known Miura-ori pattern, the crease pattern forms a tessellation of the 2D sheet by parallelograms (Fig. 1a). To obtain the 3D Miura-ori structure, the creases are folded by a combination (also known as a mountain–valley assignment) of mountain folds (folds forming a ridge) and valley folds (folds forming a trench) in a periodic manner.

Several questions then arise naturally: How is the 2D crease pattern (that is, the geometry of the folds) related to the shape of the 3D structure? How can we design the crease patterns and the mountain–valley assignments to achieve desired physical properties with complex shapes? More importantly, how can we perform the design tasks efficiently? This motivates us to consider the computational design of origami metamaterials.

To tackle the design problem, one recent approach by Dieleman et al.[1] is to first identify certain distinct foldable motifs and then use them as jigsaw pieces for origami design. Specifically, for four vertices W, X, Y, Z around a plate, the angle sum condition $\alpha^W + \beta^X + \gamma^Y + \delta^Z = 2\pi$, where $\alpha^W, \beta^X, \gamma^Y, \delta^Z$ are the four interior angles, and the loop condition $\mathbf{P}^Z_\delta \cdot \mathbf{P}^Y_\gamma \cdot \mathbf{P}^X_\beta \cdot \mathbf{P}^W_\alpha = 1$, where $\mathbf{P}^Z_\delta, \mathbf{P}^Y_\gamma, \mathbf{P}^X_\beta, \mathbf{P}^W_\alpha$ are the fold operators, should

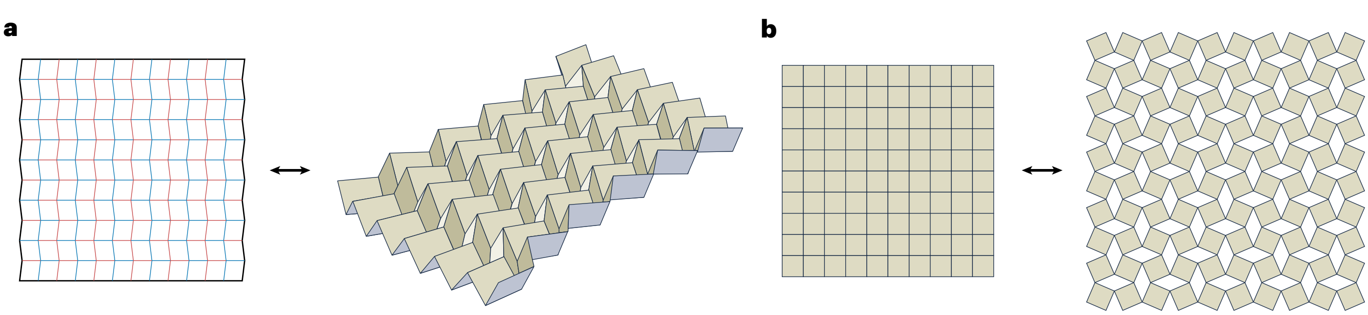

Fig. 1 | Basic origami and kirigami patterns. a, The crease pattern and the folded configuration of a standard Miura-ori pattern. The red and blue edges correspond to mountain folds and valley folds respectively. **b**, The cut pattern and the deployed configuration of a standard rotating-squares kirigami pattern.

both be satisfied. Among all possible combinations, the authors obtained 140 distinct rigidly foldable motifs. They then showed that the pieces can be fitted together to encode foldable origami patterns, and hence a large variety of origami patterns can be created by solving combinatorial problems.

Besides considering the combinations of distinct foldable motifs above, one may also start with a simple folding pattern and then 'grow' it by attaching new folds to it following some compatibility rules. Dudte et al.[2] proposed an additive design framework for designing developable quad origami patterns. To grow an existing folded quad origami model by adding new origami strips along the growth front, they identified the relationship between angles at adjacent growth-front vertices and proved that all interior design angles are uniquely determined by one flap angle sweeping from a face in the new strip about a growth-front edge. This observation allowed them to effectively characterize the design space of generic quad origami patterns and formulate a marching algorithm to create various ordered, disordered, straight-, and curved-fold origami structures from a simple folded origami seed.

Note that the key of the above work is to focus on the design space of a single-vertex origami and identify the possible angles and edge lengths, thereby enabling the design of origami structures with complex geometries. In many applications, it is also common to consider other physical properties of the origami structures. For instance, the rigid foldability, that is, the ability for the origami structure to continuously transform from the 2D state to the 3D configuration without any deformation of the facets, is an important physical property for many engineering structures involving rigid materials. In a recent work[3], Walker and Stankovic developed an algorithm based on the principle of three units (PTU), a principle for rigidly foldable degree-n origami vertices, for designing rigidly foldable origami structures. Specifically, by representing crease patterns as directed acyclic graphs and repeatedly applying the PTU, they can easily generate and optimize crease patterns for achieving different rigidly foldable origami structures. Moreover, by including additional operations such as mirroring, translating, and gluing, their approach can be used to produce 3D tessellations and other panel-hinge assemblages with a single kinematic degree of freedom. Dang et al.[4] also proposed an inverse design framework for rigidly and flat-foldable quadrilateral origami (RFFQM) structures. Specifically, their algorithm focuses on designing RFFQM structures to approximate a given 3D target shape. Because of the rigidly and flat-foldable condition, the origami structures produced by their framework can be easily transformed from a flat sheet into the desired folded state to approximate the target shape by a controlled folding motion. They can be further transformed into a compact folded state for better portability and easier storage.

Note that most of the above works have focused on quad origami patterns in which the folds are straight edges forming quadrilateral tiles. It is natural to ask whether one can create even more complex origami structures involving curved creases and curved tiles to achieve better fits to certain target shapes and enhance the mechanical stiffness of the structures. Recently, Liu and James[5] proposed a method for general curved tile origami design using a group orbit procedure. Their method starts with a curved tile origami unit cell in which the rulings are used as creases and an extra crease is introduced to gain additional freedom. Then, by repeatedly applying different Euclidean groups such as helical groups, circle groups, translation groups and conformal groups to the unit cell, a wide range of origami structures with curved tiles can be obtained.

Besides the above-mentioned algorithmic developments in origami design, having generic computational origami software systems is also important for practical design problems and industrial applications. While there are some existing software packages for origami design and analysis, most of them only involve complicated mathematical conditions and require knowledge of computer programming and hence may not be accessible to designers without the relevant mathematical and engineering background. To overcome this issue, Yu et al.[6] proposed the use of shape grammar formalism and the Shape Machine, a new shape grammar interpreter, for origami design. Specifically, by considering origami patterns as arrangements of lines and arcs, the generation of origami patterns can be done by a series of executable shape rules focusing on their visual aspects without involving explicit mathematical formulas. This approach allows the designers to produce different origami structures by visual reasoning and hence is more accessible and intuitive when compared to some prior origami design software tools. Besides, different tasks for computational origami design such as pattern design, folding simulation and physical fabrication are usually handled separately. To offer a more seamless design process for the designers, Suto et al.[7] developed an integrated platform called Crane for computational origami design. The platform accepts inputs in various forms such as 2D pattern designs, 3D origami tessellations, and arbitrary meshes. Different origami products can then be designed based on the desired functionality and foldability. Specific constraints on the symmetry, developability, fixed angles and vertex positions can be added to the design process, with the folding simulations also provided to the user. The platform supports further thickening the output 2D crease pattern into a 3D volume with hinges for different fabrication tools.

Altogether, there has been significant progress in the algorithmic and software development for computational origami design in recent years. A wide range of origami structures with different desired physical properties and shapes can be produced by the developed algorithms. However, most of the current approaches still rely on mathematical optimization and may not be efficient enough for some large-scale design problems. A possible direction would be to utilize AI for origami design, which could accelerate the design process and potentially create new classes of origami structures not found by the current approaches.

Computational kirigami design

Besides using folds, another way to create shape change for a sheet of materials is to introduce cuts in it to make it morphable. In traditional paper art, kirigami is a variation of origami in which both folds and cuts can be used to create specific 3D shapes from paper. However, in most of the prior scientific and engineering kirigami studies, kirigami has been considered as a class of techniques involving cuts only to create flexible structures with either in-plane or 2D-to-3D shape changes.

The rotating-squares kirigami pattern (Fig. 1b) is one of the most widely used cutting patterns in kirigami studies. Specifically, by introducing vertical and horizontal cuts in an alternating manner, one can form a tessellation of square tiles connected by joints in a periodic manner. The perforated sheet can then be transformed into another configuration through a continuous deployment process. As in origami design, here it is natural to ask how one can change the cut geometry, that is, the size and position of the cuts, such that the cut pattern can lead to certain shape changes and satisfy the desired physical properties such as rigid deployability (that is, the ability for the structure to morph from one state to another state with no tile strain). One may also consider changing the cut topology of the kirigami pattern to achieve rigidity or floppiness. While some existing works have explored different variants of the rotating-squares pattern such as rectangle-based, rhombus-based, triangle-based, and hexagon-based patterns with periodic cuts, designing more complex kirigami structures remains to be a challenging task. Specifically, note that the deployment of a kirigami structure involves a coordinated rotation of its tiles. Therefore, modifying the cut geometry or cut topology locally may already change the shape-morphing effect of the entire structure significantly. This makes the design of kirigami patterns with more general non-periodic cuts particularly difficult, as one would need to consider the overall effect of all such cuts on the final deployed kirigami structure. Hence, there is a great need for computational tools for a more efficient and systematic kirigami design process.

To overcome the challenge of finding an admissible cut pattern for achieving certain desired shape changes automatically, Choi et al.[8] proposed an inverse design framework for creating shape-morphing kirigami structures that can approximate different target shapes in 2D or 3D. Specifically, given a target shape and a standard kirigami pattern, the framework first identifies the geometric constraints including the angle sum constraint and the edge length constraint for guaranteeing the compatibility between the kirigami tiles. It then solves a constrained optimization problem on the vertex positions in the deployed space, with the compatibility constraints and the shape-matching constraints encoded so that the resulting deployed structure matches the target shape and admits a closed and compact contracted configuration. Note that the above approach focuses on designing cut patterns on a 2D sheet of materials without any topological holes. In practice, there may be some defects such as holes or cracks in the sheet of materials, which will change its topology and make the above approach not applicable. Therefore, it is also important to develop a method that can overcome this issue and handle the kirigami design problem for more topologically complex 2D sheets of materials. To achieve this, Dang et al.[9] later developed a design method for planar quadrilateral kirigami tessellations with different topologies. They considered introducing cuts on flat sheets with different numbers of holes to form kirigami tessellations consisting of quadrilaterals and proved a theorem for the rigid deployability of such kirigami tessellations. Then, they obtained rigidly deployable planar quadrilateral kirigami patterns by formulating the kirigami vertex positions by a system of linear equations and solving an optimization problem with the nonlinear geometrical constraints given by the theorem.

While the above-mentioned inverse design framework can handle various kirigami design problems well, the constrained optimization involves all vertex coordinates as variables and hence may be time-consuming for large-scale design problems. Also, since the constrained optimization approach encodes different desired physical properties as individual constraints, it is difficult to study the design space and limitations of the design problem in a systematic manner. To resolve this issue, Dudte et al.[10] developed an additive framework for quad kirigami design with different fundamental parameters in lengths, angles and boundary positions decoupled. Specifically, by considering the negative space of a quad kirigami unit cell as a parallelogram four-bar linkage, they established the relationship between all adjacent linkages in a linkage array via dynamic programming, from which they obtained a linkage array design matrix. The compatibility, reconfigurability and rigid deployability of quad kirigami patterns can be further encoded into the design matrix, thereby leading to a simple linear design strategy involving solving a matrix equation with prescribed boundary constraints. A large variety of compact reconfigurable and rigidly deployable kirigami patterns can then be obtained by suitably modifying the length and angle parameters independently.

Note that the kirigami design approaches we discussed so far have primarily focused on the ability to approximate a target shape, with some possible further controls of the rigid deployability of the pattern under simple mechanical loading. In many practical applications, it may also be important to consider more complex deployment or actuation processes, in which the underlying physical laws would greatly affect the feasibility of the desired shape-morphing effects. To address the physical feasibility aspect of kirigami, Wang et al.[11] proposed a framework for the physics-aware differentiable design of magnetically actuated kirigami. Specifically, they considered active kirigami embedded with magnetic particles that can achieve shape morphing upon magnetic excitation. In addition to the geometrical compatibility constraints, they also incorporated the physical equilibrium requirements into a constrained optimization problem to ensure the physical feasibility of the resulting kirigami structure.

Besides changing the cut geometry as in all the above-mentioned approaches, one may also control the degrees of freedom in the deployment process of kirigami structures by changing the cut topology. To achieve this, Chen et al.[12] proposed computational frameworks for manipulating the connections between kirigami tiles using both deterministic and stochastic approaches. Specifically, they developed a deterministic method for controlling the overall rigidity of the kirigami pattern by changing the minimum number of connections between the tiles. They have also studied the change in the floppiness of the kirigami structures with randomly added tile connections, thereby providing an alternative approach for kirigami design.

As described at the beginning of this section, most of the prior kirigami approaches in science and engineering have only considered cuts, while the original concept of kirigami in art may also involve folds. Recently, Zhang et al.[13] developed a computational approach for designing folding patterns in a kirigami structure to achieve a strong, lightweight metamaterial. They considered a 2D checkerboard kirigami pattern and an assignment of mountain–valley folds that can turn the 2D sheet into a 3D checkerboard shell. Then, to obtain a kirigami metamaterial that can be transformed into a given 3D object, they discretized the object into hexahedral elements and deployed them to a planar sheet one-by-one. They then further assigned specific folding directions and added mechanical connectors, thereby forming the foldable kirigami metamaterial. They applied this approach to create kirigami metamaterials that can morph into various curved surfaces with a high strength-to-weight ratio.

Besides, with the rapid development of machine learning in recent years, some recent works have also utilized machine learning techniques to overcome challenges in the automatic design of kirigami. For instance, due to the nonlinear effects caused by the out-of-plane buckling of graphene kirigami structures, predicting their mechanical properties is difficult. To resolve this issue, Hanakata et al.[14] used a supervised autoencoder (SAE) for the inverse structural design of graphene kirigami. Using the SAE, they were able to generate designs consisting of mixed parallel and orthogonal cuts and predict their mechanical properties. Alderete et al.[15] introduced another machine learning framework involving a tandem deep neural network architecture for the inverse design of kirigami meta-atoms. Specifically, they considered a 2D kirigami motif with a pair of internal line cuts and a pair of U-shaped cuts that can achieve out-of-plane deformations. Using the machine learning framework, they were able to predict the cut layout of the kirigami meta-atom for obtaining a target deformed configuration. They further applied the framework to the design at the metamaterial level by combining arrays of kirigami meta-atoms.

Overall, while the computational design of kirigami is still relatively new when compared to origami, it has been gaining increasing popularity in recent years, with various algorithms developed for creating a wide range of kirigami structures. However, unlike origami, there has not been much attention on the software development for kirigami design, which poses challenges for practical designers to create kirigami structures.

Outlook

The state-of-the-art computational design approaches have enabled the design of a large class of art-inspired metamaterials. Specifically, using the current techniques, one can easily design origami metamaterials ranging from quadrilateral origami tessellations to curved tile origami structures. One can also utilize the latest kirigami design algorithms to create a wide range of kirigami metamaterials to achieve certain 2D or 3D shape-morphing effects as well as other desired physical properties. More generally, these approaches can be effectively utilized for designing complex mechanical metamaterials for various applications in science and engineering.

However, it is worth noting that the origami and kirigami design problems are still considered separately in most of the current approaches, with very limited works devoted to the computational design problem involving both folds and cuts. Therefore, one promising direction for future computational design of art-inspired metamaterials is to incorporate more general combinations of folds and cuts in the computational frameworks, which would give us more degrees of freedom in the design of mechanical metamaterials.

Also, as discussed above, while recent years have witnessed unprecedented development in machine learning and artificial intelligence, their application to art-inspired metamaterial design has been limited to only a few kirigami design problems with some relatively simple shapes. Note that most of the existing computational origami and kirigami design methods involve solving some optimization problems for which the complexity increases with the number of folds and cuts. Hence, these approaches may not be efficient enough for some large-scale design problems in industrial applications. A future direction would be to further leverage data-driven approaches for the automatic design of complex art-inspired metamaterials. Ultimately, this would greatly simplify the design process and further allow us to create novel structures for a wider range of applications.

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Competing interests

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