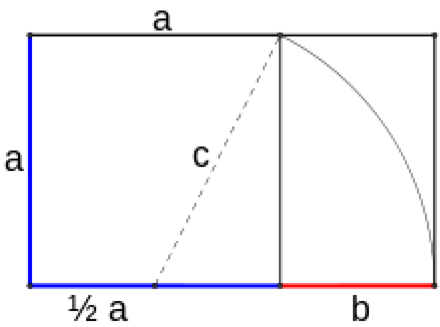
UGED 1533 - Mathematics in Visual Art

The Golden Ratio

There are a variety of perspectives from which to view the Golden Ratio φ (Greek letter "phi"). In the context of dynamic proportions, φ corresponds to the **Golden Rectangle** constructed as follows[3]:

1. Construct a simple square.
2. Draw a line from the midpoint of one side of the square to an opposite corner.
3. Use that line as the radius to draw an arc that defines the height of the rectangle.
4. Complete the golden rectangle.



The Golden Rectangle

The Golden Ratio φ is equal to the length of the longer side of this rectangle divided by that of the shorted side. By a simple application of the Pythagorean theorem, it may be derived that:

$$\varphi = \frac{a+b}{a} = \frac{1+\sqrt{5}}{2} \approx 1.6180339887\ldots$$

Observe that if you cut out a perfect square with the same height as the Golden Rectangle from one end of the rectangle, the remaining subrectangle is also a Golden Rectangle, in that the ratio of length of the longer side to that of the shorter side is equal to φ. Algebraically, this corresponds to the equality:

$$\frac{\varphi}{1} = \frac{1+\varphi}{\varphi}$$

In fact, applying the quadratic formula that we learned in secondary school, we see that φ is one of two solutions to the equation:

$$\frac{x}{1} = \frac{1+x}{x},$$

the other solution being: $\frac{1-\sqrt{5}}{2} = -\frac{1}{\varphi}$.

Another interesting fact about φ is that it is the "limit" of the ratio of consecutive terms in the Fibonacci sequence, namely the sequence where the first two terms is 1, and each successive term is equal to the sum of the previous two terms. Hence, the sequence is:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots$$

The ratio consecutive terms give the sequence:

$$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \ldots$$

which numerically gets closer to closer to φ. For example, $\frac{8}{5}$ is decimal form is 1.6, which is already quite close to $\varphi \approx 1.6180339887\ldots$.
Many people have claimed that the Golden Ratio figures in the designs of a number of ancient architectural works such as the Great Pyramid in Egypt and the Parthenon in Greece, and in the works of Leonardo da Vinci[4].

The Parthenon, 438 BCE

The Annunciation (c. 1472)
Leonardo da Vinci

Such claims have generally been met with criticisms regarding their scientific rigour[5]. In any event, that the Golden Ratio or other dynamic proportions have a special status with regard to pictorial decomposition is an idea which many artists have at least been exposed to, whether or not they (consciously or otherwise) incorporate it into their own artwork. Two alleged examples of artists who utilized dyamic proportions are American painters George Bellows (1882 - 1925) and Maxfield Parrish (1870 - 1966).

Dempsey and Firpo (1924)
George Bellows

Notions in dynamic proportions such as root rectangles are being taught in illustration courses to this very day.

1.1 References

References

1. Tobin, Richard. *The Canon of Polykleitos*, American Journal of Archaeology in 1975, Vol. 79, No. 4 (Oct., 1975), pp. 307-321.
2. Hambidge, Jay. *The Elements of Dynamic Symmetry*, Dover Publications (June 1, 1967).
3. https://en.wikipedia.org/wiki/Golden_rectangle
4. http://www.goldennumber.net/category/design/
5. Markowsky, George. *Misconceptions about the Golden Ratio*, The College Mathematics Journal Vol. 23, No. 1 (Jan., 1992), pp. 2-19 JSTOR