THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
UGEB2530C Games and Strategic Thinking (2024 Spring)
Homework 2
Due date: 23:59 on April 7

Name: ________________________________ Student No.: __________________

I declare that the assignment here submitted is original except for source material explicitly acknowledged, the piece of work, or a part of the piece of work has not been submitted for more than one purpose (i.e. to satisfy the requirements in two different courses) without declaration, and that the submitted soft copy with details listed in the "Submission Details" is identical to the hard copy, if any, which has been submitted. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained on the University website https://www.cuhk.edu.hk/policy/academichonesty/

It is also understood that assignments without a properly signed declaration by the student concerned will not be graded by the course teacher.

______________________________ ______________________________
Signature Date

General Guidelines for Homework Submission.

- Please submit your answer as a single pdf file to Gradescope through the course UGEB2530B in Blackboard.
- In Gradescope, for each question, please indicate exactly which page(s) its answer locates. **Answers of incorrectly matched questions will not be graded**.
- **Late submission will NOT be graded and result in zero score**. Any answers showing evidence of plagiarism will also score zero; stronger disciplinary action may also be taken.
- Points will only be awarded for answers with sufficient justifications.

1. Find all the pure Nash equilibria of the games with the following game bimatrices and state whether they are Pareto optimal.

   (a) $\begin{pmatrix} (1,3) & (4,6) \\ (2,5) & (1,3) \end{pmatrix}$

   (b) $\begin{pmatrix} (-1,2) & (3,6) & (1,-3) \\ (3,1) & (5,-1) & (4,2) \\ (6,3) & (-2,2) & (3,0) \end{pmatrix}$

2. Consider the 2-person game with the following bimatrix

$$\begin{pmatrix} (1,4) & (6,1) \\ (4,3) & (2,5) \end{pmatrix}$$

   (a) Find the prudential strategy of each player and the security levels of the players.

   (b) Find a Nash equilibrium of the game and the corresponding payoffs to the players.

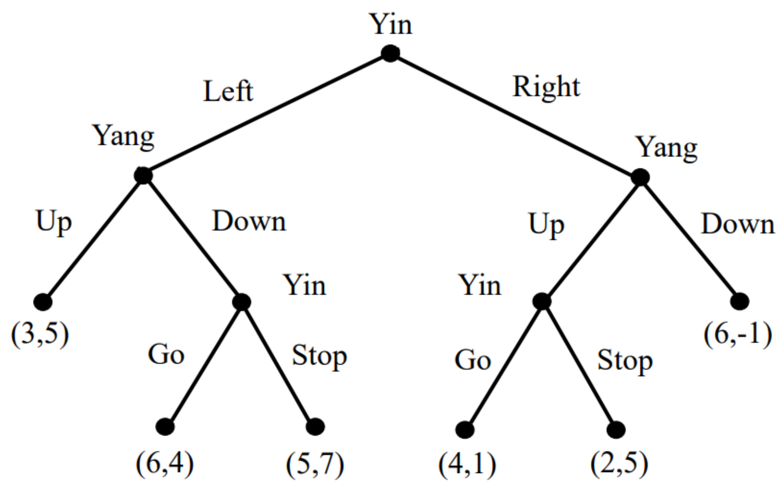
3. The battle between a batsman and a bowler in a cricket game just reached the head-to-head competition stage. The bowler is either taking an inswing or an outswing action, while the batsman is either using an offside or a legside action. If the bowler uses inswing and the batsman uses offside, the profits for the bowler and the batsman will be 5 and 5 respectively. If the bowler uses inswing and the batsman uses legside, the profits for the bowler and the batsman will be 9 and 12 respectively. If the bowler uses outswing and the batsman uses offside, the profits for the bowler and the batsman will be 13 and 11 respectively. If the bowler uses outswing and the batsman uses legside, the profits for the bowler and the batsman will be 7 and 8 respectively.

   (a) Write down the game bimatrix. (Use the bowler as the row player)

   (b) Find the prudential strategies and the security levels of the two competitors.

   (c) Given that a non-pure Nash equilibrium exists, find all the Nash equilibria and the corresponding payoffs to two competitors.

4. Find a Nash equilibrium of the game with the following game tree by backward induction and write down the corresponding payoff pair.

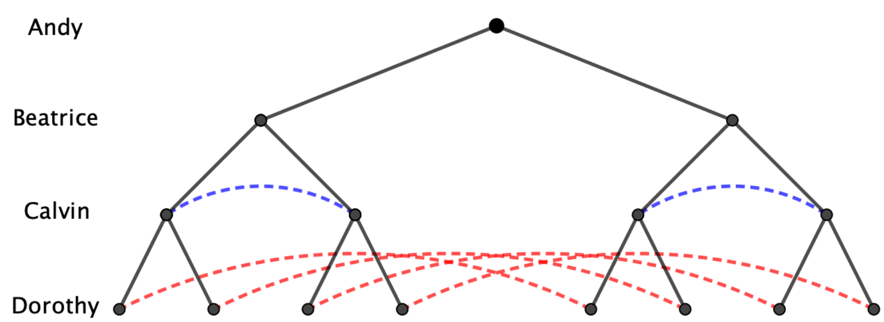


5. Mary and Abe start with \$10 in each of their piles. They take turns choosing one of two actions, continue or stop, with Mary choosing first. Each time a player says continue, \$10 will be removed from her pile, and \$20 will be added to the other player's pile. The game automatically stops when the total amount in their piles reaches \$60.

   (a) Draw the game tree of the game.

   (b) Find a Nash equilibrium of the game by backward induction and write down the corresponding payoffs.

6. Consider the following game tree:



Circle the correct answers.

- **Beatrice** knows / doesn't know **Andy**'s choice.
- **Calvin** knows / doesn't know **Andy**'s choice.
- **Calvin** knows / doesn't know **Beatrice**'s choice.
- **Dorothy** knows / doesn't know **Andy**'s choice.
- **Dorothy** knows / doesn't know **Beatrice**'s choice.
- **Dorothy** knows / doesn't know **Calvin**'s choice.

7. Players I and II play the following bluffing game. Each player bets \$1. Player I is given a card which is high or low; each is equally likely. Player I sees the card, player II doesn't. Player I can raise the bet to \$2 or fold. If player I folds, player I loses \$1 to player II. If player I raises, player II can call or fold. If player II folds, he loses \$1 to player I no matter what the card is. If player II calls, then player I wins \$2 from player II if his card is high and loses \$2 to player II if the card is low.

   (a) Draw the game tree of the game.

   (b) Write down all strategies of players I and II.

   (c) Write down the strategic form (game matrix) of the game.

   (d) Solve the game.

8. Anna has two coins. One is fair (probability 1/2 of heads and 1/2 of tails) and the other is biased with probability 1/4 of heads and 3/4 of tails. Anna knows which coin is fair and which is biased. She selects one of the coins and tosses it. The outcome of the toss is announced to Elsa. Then Elsa must guess whether Anna chose the fair or biased coin. If Elsa is incorrect, she pays $2 to Anna, and if she is correct, she receives $2 from Anna.

   (a) Draw the game tree.

   (b) Write down all strategies of Anna and Elsa.

   (c) Write down the strategic form of the game.

   (d) Solve the game.

Extra exercises

E1. For the following 2-person simultaneous game,

	C1	C2
R1	(5,1)	(-4,0)
R2	(2, 4)	(-2,7)

(a) Find all the pure Nash equilibrium(s) (if any exist) and the corresponding payoffs. Determine if each of them is Pareto optimal.

(b) Given that a non-pure Nash equilibrium exists, find it and the corresponding payoffs.

(c) Find **Row**'s prudential strategy and security level.

(d) Find **Col**'s prudential strategy and security level.

E2. Consider the following modified Chicken game:

	Swerve	Don't
Swerve	(0,0)	(-3,2)
Don't	(3,-2)	(-10,-10)

Col cares about winning/losing face, but **Row** cares even more.

(a) Find all the pure Nash equilibrium(s) (if any exist) and the corresponding payoffs.

(b) Given that a non-pure Nash equilibrium exists, find it and the corresponding payoffs.

E3. For the following 2-person simultaneous game,

	C1	C2
R1	(5,-3)	(2,4)
R2	(1,3)	(-1,0)

(a) Find all the pure Nash equilibrium(s) (if any exist) and the corresponding payoffs.

(b) Find **Row**'s prudential strategy and his/her security level.

(c) Find **Col**'s prudential strategy and his/her security level.

E4. Let us consider the 2-person game used by a non-profit organisation (social hut) that wishes to aid a needy man if he looks for work but not if he does not try. The payoffs are $5, -2$ (for organisation, needy man) if the organisation aids and the needy man tries to work; $1, 3$ if the organisation does not aid and the needy man tries to work; $4, 2$ if the organisation aids and the needy man does not try to work; and $-3, 0$ in the remaining case.

(a) Write down the game bimatrix. (Use organisation aid as the row player)

(b) Find the prudential strategy of each player and the security levels of the players.

(c) Find a Nash equilibrium of the game and the corresponding payoffs to the players.

E5. Two technology companies A and B plan to produce two 5G (Fifth-generation wireless) products. Each of the companies may produce either a 5G modem or a 5G New Radio millimeter. If both companies produce a 5G modem, each of the companies would have a profit of \$5 million. If company A produces a 5G modem and company B produces a 5G New Radio millimeter, the profits of A and B will be \$9 million and \$12 million respectively. If A produces a 5G New Radio millimeter and B produces a 5G modem , the profits of A and B will be \$13 million and \$11 million respectively. If both companies produce a 5G New Radio millimeter, the profits of A and B will be \$7 million and \$8 million, respectively.

(a) Write down the game bimatrix. (Payoffs to A and B in millions of dollars. Company A, B as row, column players respectively)

(b) Find the prudential strategies and the security levels of the two companies.

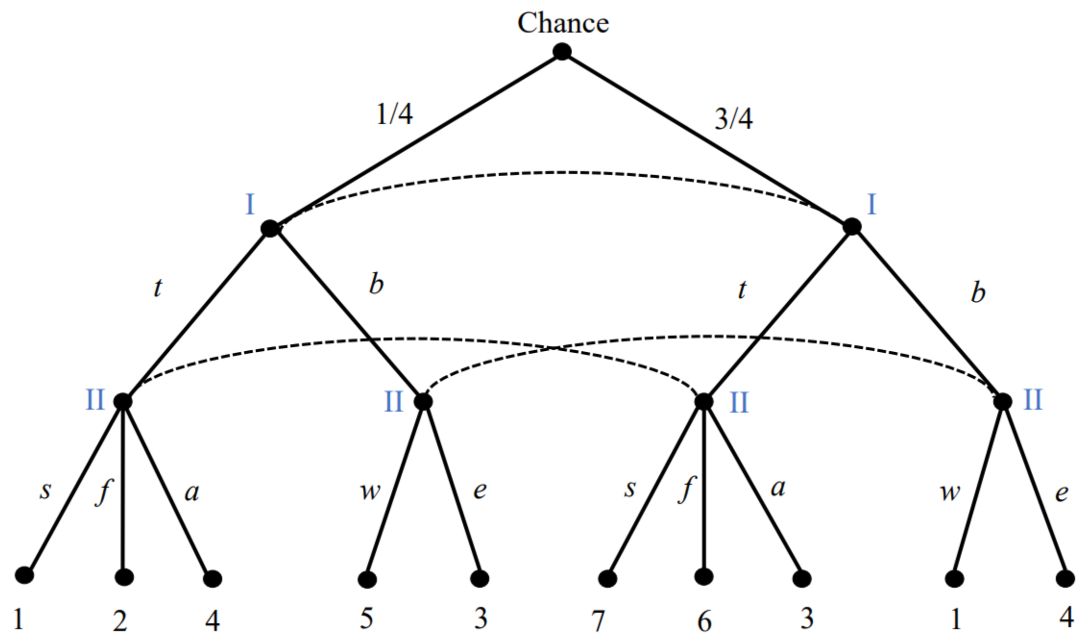
(c) Given that a non-pure Nash equilibrium exists, find all the Nash equilibria and the corresponding payoffs to the two companies.

E6. Consider the following game. **Harden** and **James** are both in room 1 and **Curry** is in room 2. **Harden** first flips a coin. **James** goes to room 2 afterward and tells **Curry** *Head* or *Tail*. **Curry** then chooses to *Believe*, *Not* or *Fold*.

- **James** pays **Curry** \$10 if what **James** said matches the flipped coin and **Curry** chooses *Believe*
- **James** pays **Curry** \$20 if what **James** said doesn't match the flipped coin and **Curry** chooses *Not*
- **Curry** pays **James** \$20 if what **James** said matches the flipped coin and **Curry** chooses *Not*
- **Curry** pays **James** \$10 if what **James** said doesn't match the flipped coin and **Curry** chooses *Believe*
- **Curry** pays **James** \$2 if **Curry** chooses to *Fold*

Draw its game tree using **Harden**, **James**, **Curry** as player $1, 2, 3$ including **James**' payoff for each outcome.

E7. Consider a zero sum game between Player I and Player II with game tree



Here s denotes spirit, f denotes fire, a denotes air, w denotes water, e denotes earth, t denotes top, and b denotes bottom.

The numbers assigned to the terminal nodes are the payoffs to Player I.

(a) Write down all strategies of Player I and Player II.

(b) Find the strategic form (game matrix) of the zero sum game.

E8. Mickey chooses a number from 5, 6 and 7. Minnie, who knows whether the chosen number is odd or even but does not know the exact value, must choose between 2 or 3. Then Minnie pays Mickey with an amount equal to the (absolute) difference of the numbers.

(a) Draw the game tree of the game.

(b) Write down all strategies of Mickey and Minnie.

(c) Find the maximin strategy of Mickey, the minimax strategy of Minnie and the value of the game.