

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MMAT5620:Mathematics Enhancement for Teachers 2023-2024 Term 1
Homework Assignment 3
Due Date: 15 December 2023 (Friday) before 11:59 PM

I declare that the assignment here submitted is original except for source material explicitly acknowledged, the piece of work, or a part of the piece of work has not been submitted for more than one purpose (i.e. to satisfy the requirements in two different courses) without declaration, and that the submitted soft copy with details listed in the “Submission Details” is identical to the hard copy, if any, which has been submitted. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained on the University website <https://www.cuhk.edu.hk/policy/academichonesty/>

It is also understood that assignments without a properly signed declaration by the student concerned will not be graded by the course teacher.

Signature

Date

General Regulations

- All assignments will be submitted and graded on Gradescope. You can view your grades and submit regrade requests there as well. For submitting your PDF homework on Gradescope, [here are a few tips](#).

Where is Gradescope?

Do the following:

1. Go to [2023R1 Mathematics Enhancement for Teachers \(MMAT5620\)](#)
 2. Choose Tools in the left-hand column
 3. Scroll down to the bottom of the page
 4. The green Gradescope icon will be there
- Late assignments will receive a grade of 0.
 - Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your assignments). Please write your answers using a black or blue pen, NOT any other color or a pencil.

For the declaration sheet:

Either

Use the attached file, sign and date the statement of Academic Honesty, convert it into a PDF and submit it with your homework assignments via [Gradescope](#).

Or

Write your name on the first page of your submitted homework, and simply write out the sentence “I have read the university regulations.”

- Write your solutions on A4 white paper or use an iPad or other similar device to present your answers and submit a digital form via Gradescope. Please do not use any colored paper and make sure that your written solutions are a suitable size (easily read). Please be aware that you can only use a ball-point pen to write your answers for any exams.
- Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your answers getting good marks on this homework. Neatness and organization are also essential.

Please attempt to solve all the problems. Your solutions for problems 1-8 are to be submitted.

1. Find the domain and range of the following function. Check your answers using your own computer package/software or calculator.

$$f(x) = -\arccos\left(\frac{x}{5}\right) + \frac{\pi}{2}.$$

2. Find the inverse function f^{-1} of the following function f . State the domain and the range of f and f^{-1} .

$$f(x) = \sin(2x) \quad \text{for} \quad -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}.$$

3. Prove that each of the following equations is an identity.

(a) $1 - \sec x \cos^3 x = \sin^2 x$;

(b) $\frac{1}{\csc x - \cot x} = \frac{1 + \cos x}{\sin x}$;

(c) $\frac{\sin^2 x + \sin x - 2}{2 \sin x - 2} = \frac{\sin x + 2}{2}$;

(d) $\ln |\csc x + \cot x| = -\ln |\csc x - \cot x|$.

4. ~~The equations $f_1(x) = f_2(x)$ is an identity if and only if the graphs of $y = f_1(x)$ and $y = f_2(x)$ coincide at all values of x for which both sides are defined. For each question, either prove that it is an identity or prove that it is not an identity.~~

Establish the following identities:

(a) $\frac{\sin x + \cos x}{\sin x} = 1 + \cot x$;

(b) $\cot x + \sin x = \frac{1 + \cos x - \cos^2 x}{\sin x}$;

(c) $\frac{\sin 3x - \sin x}{\cos 3x + \cos x} = \tan x$;

(d) $\cos^2(x - y) - \cos^2(x + y) = \sin^2(x + y) - \sin^2(x - y)$.

5. Find $\sin \alpha$, $\cos \alpha$, $\tan \alpha$, $\csc \alpha$, $\sec \alpha$ and $\cot \alpha$., where

$$\cos(2\alpha) = 3/5 \quad \text{and} \quad 0^\circ < 2\alpha < 90^\circ.$$

6. Find the exact value of $\cos(2\alpha)$, given that $\sin \alpha = 8/17$ and α in quadrant II.

7. Find all real numbers in the interval $[0, 2\pi)$ that satisfy the following equation. Round approximate answers to the nearest tenth.

$$5 \sin^2 x - 2 \sin x = \cos^2 x.$$

8. Find all real numbers that satisfy the following equation, expressing the general solutions in terms of radians and let k (say) be an integer.
~~Round approximate answers to the nearest tenth.~~

$$\sin x \cos(\pi/4) + \cos x \sin(\pi/4) = 1/2.$$