THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MMAT5620:Mathematics Enhancement for Teachers 2023-2024 Term 1
Homework Assignment 1
Due Date: 13 October 2023 (Friday) before 11:59 PM

I declare that the assignment here submitted is original except for source material explicitly acknowledged, the piece of work, or a part of the piece of work has not been submitted for more than one purpose (i.e. to satisfy the requirements in two different courses) without declaration, and that the submitted soft copy with details listed in the "Submission Details" is identical to the hard copy, if any, which has been submitted. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained on the University website https://www.cuhk.edu.hk/policy/academichonesty/

It is also understood that assignments without a properly signed declaration by the student concerned will not be graded by the course teacher.

Signature ____________________ Date ____________________

General Regulations

- All assignments will be submitted and graded on Gradescope. You can view your grades and submit regrade requests there as well. For submitting your PDF homework on Gradescope, here are a few tips.

  Where is Gradescope?

  Do the following:

  1. Go to 2023R1 Mathematics Enhancement for Teachers (MMAT5620)
  2. Choose Tools in the left-hand column
  3. Scroll down to the bottom of the page
  4. The green Gradescope icon will be there

- Late assignments will receive a grade of 0.

- Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your assignments). Please write your answers using a black or blue pen, NOT any other color or a pencil.

  For the declaration sheet:

  Either

Use the attached file, sign and date the statement of Academic Honesty, convert it into a PDF and submit it with your homework assignments via Gradescope.

Or

Write your name on the first page of your submitted homework, and simply write out the sentence "I have read the university regulations."

- Write your solutions on A4 white paper or use an iPad or other similar device to present your answers and submit a digital form via Gradescope. Please do not use any colored paper and make sure that your written solutions are a suitable size (easily read). Please be aware that you can only use a ball-point pen to write your answers for any exams.

- Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your answers getting good marks on this homework. Neatness and organization are also essential.

Please attempt to solve all the problems. Your solutions for problems 1-10 are to be submitted.

We strongly recommended that you study Extra Exercises 1-6, though you are not required to submit their solutions. Suggested solutions for all the problems will be provided.

1. Use the method of successive differences to list the next three terms to continue the pattern in each of the following.

   (a) $3, 14, 31, 54, 83, 118, \cdots$

   (b) $1, 11, 35, 79, 149, 251, \cdots$

2. Answer each question independently.

   (a) Use inductive reasoning to predict the number of distinct regions in a Venn diagram with n overlapping circles.

   (b) Try to draw a Venn diagram with four overlapping circles, and confirm the result from (a) for $n = 4$. Please do it with circles.

3. Let

$$U = \{a, b, c, d, e, f, g\},\ X = \{a, c, e, g\}$$
$$Y = \{a, b, c\} \text{ and } Z = \{b, c, d, e, f\}$$

   Find

   (a) $X' \cap Y'$

   (b) $(Y \cap X') \cup Z'$

   (c) $X \cap (X - Y)$

4. Answer each question independently.

   (a) Use Venn diagrams to determine whether the following statements are equal for all sets A, B, and C.

$$\begin{array}{lll} \text{(i)} & A \cup (B \cap C)' & A' \cap (B' \cup C) \\ \text{(ii)} & (A \cup B) \cap (B \cup C) & B \cup (A \cap C) \end{array}$$

   (b) Draw a Venn diagram and use the given information to fill in the number of elements in each region.

   (i) $n(A \cup B) = 15, n(A \cap B) = 8, n(A) = 13, n(A' \cup B') = 11$

   (ii) $n(A) = 57, n(A \cap B) = 35, n(A \cup B) = 81, n(A \cap B \cap C) = 15,$ $n(A \cap C) = 21, n(B \cap C) = 25, n(C) = 49, n(B') = 52$

5. The 65 students in a classical music class were polled with the following results.

37	like Wolfgang Amadeus Mozart.
36	like Ludwig van Beethoven.
31	like Franz Joseph Haydn.
14	like Mozart and Beethoven.
21	like Mozart and Haydn.
14	like Beethoven and Haydn.
8	like all three composers.

How many of these students like:

(a) none of these composers?

(b) Mozart, but neither Beethoven nor Haydn?

(c) no more than two of these composers?

6. A music class of five guitarists and four drummers is having a recital. If each member is to perform once, how many ways can the program be arranged in each of the following cases? (Use the fundamental counting principle.)

(a) All guitarists must perform first.

(b) A guitarist must perform first, and a drummer must perform last.

(c) The entire program will alternate between guitarists and drummers.

7. Answer each question independently.

(a) Evaluate:

(i) ${}_{17}P_4$

(ii) ${}_{14}C_6$

(b) A menu has five appetizers, three soups, seven main courses, six salad dressings, and eight desserts.

(i) In how many ways can a full meal be chosen?

(ii) In how many ways can a meal be chosen if either an appetizer or a soup is ordered, but not both?

(c) Twenty players compete in a tournament.

(i) In how many ways can rankings be assigned to the top five competitors?

(ii) In how many ways can the best five competitors be chosen (without being in any particular order)?

8. Find the domain of each function:

   (a) $f(x) = \sqrt{1-x}$;

   (b) $f(x) = \dfrac{x}{\sqrt{x-4}}$.

9. For the given functions f and g, find the following. For parts (a) - (d), also find the domain.

(a)	$(f+g)(x)$	(b)	$(f-g)(x)$
(c)	$(f \cdot g)(x)$	(d)	$\left(\dfrac{f}{g}\right)(x)$

   i $f(x) = \sqrt{x}$; $g(x) = 3x - 5$;

   ii $f(x) = \dfrac{2x+3}{3x-2}$; $g(x) = \dfrac{4x}{3x-2}$.

10. Given $f(x) = 1 + \dfrac{1}{x}$ and $g(x) = \dfrac{1+x}{1-x}$. Find each composite function and find its domain:

(a)	$(f \circ g)(x)$	(b)	$(g \circ f)(x)$
(c)	$(f \circ f)(x)$	(d)	$(g \circ g)(x)$

Extra Exercises

1. The digit farthest to the right in a counting number is called the ones or units digit, because it tells how many ones are contained in the number when grouping by tens is considered. What is the ones (or units) digit in 2^{4000}?

2. Sports 24/7 runs a youth basketball program. On the first day of the season, 60 young women showed up and were categorized by age level and by preferred basketball position, as shown in the following table.

		Position			Totals
		Guard (G)	Forward (F)	Center (N)	
Age	Junior Secondary (J)	9	6	4	19
	Senior Secondary (S)	12	5	9	26
	College (C)	5	8	2	15
Totals		26	19	15	60

Using the set labels (letters) in the table, find the number of players in each of the following sets.

(a) $J \cap G$

(b) $N \cup (S \cap F)$

3. Show that the for any positive integers n and r with $r \leq n$.

(a)

$$_nC_r = {_nC_{n-r}}$$

(b)

$$_nC_{r-1} + {_nC_r} = {_{n+1}C_r}$$

4. Answer each question independently.

(a) How many of the possible 5-card hands from a standard 52-card deck would consist of the following cards?

(i) four hearts and one non-heart

(ii) two face cards and three non-face cards

(iii) two black cards, two hearts, and a diamond

(b) If a single card is drawn from a standard 52-card deck, in how many ways could it be the following? (Use the additive principle.)

(i) a club or a jack

(ii) a face card or a black card

5. Consider the following function:

$$f(x) = \frac{2x^2}{x^4+1}.$$

Answer the following questions:

(a) Is the point $(-1, 1)$ on the graph of f?

(b) If $x = 2$, what is $f(x)$? What point is on the graph of f?

(c) If $f(x) = 1$, what is x? What point(s) are on the graph of f?

(d) What is the domain of f?

(e) List the $x-$intercepts, if any, of the graph of f.

(f) List the $y-$intercepts, if there are any, of the graph of f.

6. Consider the following functions:

(a) $f(x) = \begin{cases} x+3 & \text{if } -2 \le x < 1; \\ 5 & \text{if } x = 1; \\ -x+2 & \text{if } x > 1. \end{cases}$

(b) $f(x) = \begin{cases} |x| & \text{if } -2 \le x < 0; \\ x^3 & \text{if } x > 0. \end{cases}$

Answer the following questions:

(a) Find the domain of each function.

(b) Locate any intercepts.

(c) Graph each function.

(d) Based on the graph, find the range.

(e) Is f continuous on its domain?