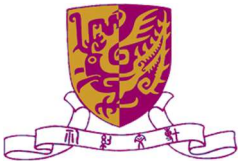


The Chinese University of Hong Kong

香港中文大學

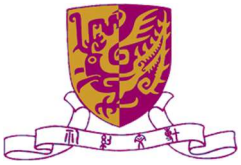
DEPARTMENT OF MATHEMATICS
THE CHINESE UNIVERSITY OF HONG KONG

Workshop on Inverse Problems for Partial Differential Equations

Abstract book

June 13-14, 2025

The Chinese University of Hong Kong



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Organizers:

Bangti Jin (b.jin@cuhk.edu.hk), The Chinese University of Hong Kong

Yavar Kian (yavar.kian@univ-rouen.fr), University of Rouen

Avenue:

Avenue: LT9, Yasumoto International Academic Park

Sponsor:

Department of Mathematics, The Chinese University of Hong Kong

List of invited speakers:

Xinlin Cao	Hong Kong Polytechnic University
Guozhi Dong	Central South University
Guanghui Hu	Nankai University
Pu-Zhao Kow	National Chengchi University
Bowen Li	City University of Hong Kong
Peijun Li	Chinese Academy of Sciences
Mikyoung Lim	Korea Advanced Institute of Science and Technology
Hongyu Liu	City University of Hong Kong
Shuai Lu	Fudan University
John Schotland	Yale University
Faouzi Triki	University of Grenoble
Haibing Wang	Southeast University
Xiang Xu	Zhejiang University
Hai Zhang	Hong Kong University of Science and Technology
Ye Zhang	Moscow-BIT Shenzhen
Ting Zhou	Zhejiang University
Zhi Zhou	Hong Kong Polytechnic University

Schedule for June 13

Time	Speaker	Title	Chair
8:55 – 9:00	Open remark		
9:00 - 9:40	Faouzi Trik	On optimal stability estimates for the multi-frequency inverse source problem	Bangti Jin
9:40 - 10:20	Peijun Li	Stability for inverse random source problems	
10:20-10:50	Coffee break		
10:50 -11:30	Guanghui Hu	Time-domain and frequency-domain methods to inverse moving point source problems	Yavar Kian
11:30-12:10	Haibing Wang	Inverse problems for diffusion equations	
12:15-14:00	Lunch at Chung Chi Staff Canteen		
14:00-14:40	Shuai Lu	Interpolation and inverse problems in spectral Barron spaces	Peijun Li
14:40-15:20	Mikyoung Lim	Designing neutral inclusions by imperfect interfaces	
15:20-16:00	Guozhi Dong	Learning based regularization in quantitative MRI	
16:00-16:30	Coffee break		
16:30-17:10	Xiang Xu	Recent stability results for inverse potential problems	Shuai Lu
17:10-17:50	Ye Zhang	Internal layer solutions and coefficient recovery in time-periodic reaction-diffusion-advection equations	
18:30-20:00	Banquet at Sai Kung		

Schedule for June 14

Time	Speaker	Title	Chair
9:00 - 9:40	John Schotland	Nonlocal PDEs and quantum optics	Hai Zhang
9:40 -10:20	Ting Zhou	Inverse problems for non-linear fractional magnetic Schrodinger equations	
10:20-10:50	Coffee break		
10:50-11:30	Hongyu Liu	Pattern formations of coupled PDEs with transparent boundary conditions and applications	Guanghui Hu
11:30-12:10	Xinlin Cao	Electromagnetic waves generated by a hybrid dieletric-plasmonic dimer	
12:15-14:00	Lunch at Chung Chi College Staff Canteen		
14:00-14:40	Hai Zhang	A theory of computational resolution limit for GMMs	Hongyu Liu
14:40-15:20	Pu-Zhao Kow	Stability of linear inverse problems: a singular value decomposition approach	
15:20-15:50	Coffee break		
15:50-16:30	Zhi Zhou	Error analysis for the numerical reconstruction of conductivity in elliptic equations	Faouzi Triki
16:30-17:10	Bowen Li	Lifting quantum Markov dynamics to speed up mixing	
17:10-18:00	Free discussions		
18:00-20:00	Dinner at Chung Chi College Staff Canteen		

Electromagnetic Waves Generated by a Hybrid Dieletric-Plasmonic Dimer

Xinlin Cao

Hong Kong Poly University, xinlin.cao@polyu.edu.hk)

Abstract: In this talk, we are concerned with the electromagnetic wave generated by a hybrid dimer (composed of two closely coupled nano-particles with one plasmonic and the other dielectric), which can polarize both the incident electric and magnetic fields. Consequently, such hybrid dimers have the potential to modify both the electric permittivity and magnetic permeability of the surrounding medium if the two nano-particles share common resonant frequencies. We derive the asymptotic expansion of the fields generated by these hybrid dimers in the subwavelength regime for incident frequencies near their shared resonant frequencies.

Learning based regularization in quantitative MRI

Guozhi Dong (guozhi.dong@csu.edu.cn)

Central South University

Abstract: N.A.

Time-domain and frequency-domain methods to inverse moving point source problems

Guanghui Hu (ghhu@nankai.edu.cn)

Nankai University

Abstract: This talk is concerned with uniqueness, stability and algorithms for inverse moving point source problems modeled by the acoustic wave equation. The purpose is to recover the orbit of a moving point source from the dynamical data recorded at a finite number of observation points. In the time domain, we derive an ordinary differential equation for the distance function between an observation point and the moving target. Solving such ODEs at four observation points yields the orbit function of the moving source. The frequency-domain method is to Fourier-transform the time-dependent source problem of the wave equation into an equivalent source problem of the Helmholtz equation with multi-frequency near-field data. This turns out to be a special wavenumber-dependent inverse source problem in the time-harmonic regime. We shall discuss the concept of non-observation directions and a non-iterative approach for imaging the orbit function. A comparison of the time-domain and frequency-domain method will be remarked at the end of the talk.

Stability of linear inverse problems: a singular value decomposition approach

Pu-Zhao Kow (pzkow@g.nccu.edu.tw)

National Chengchi University

Abstract: The main theme of this talk is to explain a general principle that the stability of linear inverse problem can be known from the decay asymptotic of singular values. For simplicity, we consider the problem of recovering the density of a Herglotz wave function, based on [1]. We also exhibit some results in [2]. which is a part of Helsinki Speech Challenge 2024 [3].

[1] Pu-Zhao Kow, Mikko Salo and Sen Zou, *Increasing resolution and instability for linear inverse scattering problems*, J. Funct. Anal. (2025) https://doi.org/10.1016/j.jfa.2025.110923 https://arxiv.org/abs/2404.18482

[2] Pu-Yun Kow and Pu-Zhao Kow, *An efficient light-weighted signal reconstruction method consists of Fast Fourier Transform and Convolutional-based Autoencoder*, arXiv preprint (2024) https://doi.org/10.48550/arXiv.2501.01650

[3] Martin Ludvigsen, Elli Karvonen, Markus Juvonen and Samuli Siltanen, *Helsinki Speech Challenge*, arXiv preprint (2024) https://doi.org/10.48550/arXiv.2406.04123

Lifting quantum Markov dynamics to speed up mixing

Bowen Li (bowen.li@cityu.edu.hk)

City University of Hong Kong

Abstract: We generalize the concept of non-reversible lifts for reversible diffusion processes initiated by Eberle and Lorler (2024) to quantum Markov dynamics. The lifting operation, which naturally results in hypocoercive processes, can be formally interpreted as, though not restricted to, the reverse of the overdamped limit. We prove that the L^2 convergence rate of the lifted process is bounded above by the square root of the spectral gap of its overdamped dynamics, indicating that the lifting approach can at most achieve a transition from diffusive to ballistic mixing speeds. Further, using the variational hypocoercivity framework based on space-time Poincare inequalities, we derive a lower bound for the convergence rate of the lifted dynamics. These findings offer quantitative convergence guarantees for hypocoercive quantum Markov processes, and also characterize the potential and limitations of accelerating the convergence through lifting. As applications, we construct optimal lifts for various detailed balanced classical and quantum processes, including the symmetric random walk on a chain, the depolarizing semigroup, Schur multipliers, and quantum Markov semigroups on group von Neumann algebras.

Stability for inverse random source problems

Peijun Li (lipeijun@lsec.cc.ac.cn)

Chinese Academy of Sciences

Abstract: In the field of inverse problems, the estimation of an unknown source term from indirect observations is a fundamental challenge. Random sources add another level of complexity to this problem due to their uncertainties. In this talk, we will focus on the stability estimates for inverse random source problems of wave equations. An overview will be provided on the existing results for estimating the stability of the solution in deterministic settings, and our recent findings will be presented for the stochastic cases.

Designing neutral inclusions by imperfect interfaces

Mikyoung Lim (hanmklim@kaist.ac.kr)

Korea Advanced Institute of Science and Technology

Abstract: In this talk, we investigate the problem of a planar conductivity inclusion with the imperfect interface condition, where the inclusion is simply connected. Using the layer potential approach and geometric function theory on complex functions, we derive explicit matrix expressions for the generalized polarization tensors (GPTs), which are the expansion coefficients for the perturbation in the incident background field due to the inclusion. Using these expressions, we construct GPT-vanishing structures consisting of a simply connected core with an imperfect interface that yield negligible perturbations for uniform incident fields. Additionally, we explore the neutral inclusion problem with the imperfect interface condition via physics-informed neural networks (PINNs), a powerful tool for simultaneously solving forward and inverse problems governed by partial differential equations. Notably, the neutrality assumption serves as training data for the neural networks.

Pattern formations of coupled PDEs with transparent boundary conditions and applications

Hongyu Liu (hongyliu@cityu.edu.hk)

City University of Hong Kong

Abstract: In this talk, I will discuss our study on a class of coupled PDEs with transparent boundary conditions that arises in a variety of cutting-edge applications including inverse boundary problems, spectral geometry of invisibility, and super-resolution imaging. I shall report several peculiar local and global pattern formations of these PDEs. Several novel applications will also be discussed.

Interpolation and inverse problems in spectral Barron spaces

Shuai Lu (slu@fudan.edu.cn)

Fudan University

Abstract: Spectral Barron spaces, which quantify the absolute value of weighted Fourier coefficients of a function, have gained considerable attention due to their capability for universal approximation across certain function classes. By establishing a connection between these spaces and a specific positive linear operator, we investigate the interpolation and scaling relationships among diverse spectral Barron spaces. Furthermore, we introduce a link condition by relating the spectral Barron space to inverse problems, illustrating this with three exemplary cases. We revisit the notion of universal approximation within the context of spectral Barron spaces and validate an error bound for Tikhonov regularization, penalized by the spectral Barron norm. It is a joint work with Peter Mathé (WIAS).

Recent stability results for inverse potential problems

Xiang Xu (xxu@zju.edu.cn)

Zhejiang University

Abstract: In this talk, we will discuss two recent stability results for inverse potential problems. Considering an inverse elastic potential problem, we aims to reconstruct the potential utilizing the DtN map. For isotropic potential, we have derived a result of increasing stability, which consists of two parts: a Holder type, and a logarithmic part that vanishes as the frequency increases. For anisotropic potential, by constructing different pairs of real and complex exponential solutions, we have derived a similar increase in stability for the linearized inverse problem. Moreover, based on the linearized problem, we proposed a reconstruction algorithm to recover the Fourier coefficients of the potential function's elements and verified the effectiveness of the proposed algorithm by numerical examples. Furthermore, we consider a biharmonic Schrodinger operator, aiming to reconstruct the first-order perturbation term from the far-field data. A stability estimate for determining the divergence-free of the first-order perturbation A through far-field data at multiple wavenumbers. Moreover, a similar algorithm is proposed to compute the Fourier coefficients. Numerical examples are conducted to verify the effectiveness of the algorithm.

Nonlocal PDEs and quantum optics

John Schotland (john.schotland@yale.edu)

Yale University

Abstract: Quantum optics is the quantum theory of the interaction of light and matter. This talk will present a survey of recent results on related many-body problems involving the propagation of entangled photons in systems of two-level atoms. In this setting, there is a close relation to kinetic equations for nonlocal PDEs with random coefficients.

On optimal stability estimates for the multi-frequency inverse source problem

Faouzi Triki (faouzi.triki@univ-grenoble-alpes.fr)

University of Grenoble

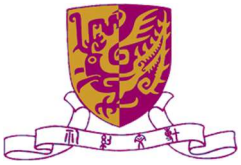
Abstract: N.A.

Inverse Problems for diffusion equations

Haibing Wang (hbwang@seu.edu.cn)

Southeast University

Abstract: In this talk, we show our recent works on inverse problems for diffusion equations. First, we consider an inverse boundary value problem. Two domain sampling methods, namely, the range test and no-response test are developed, and their convergences are proved. Second, we consider an inverse source problem with boundary measurement. A non-iterative reconstruction method, based on the range test idea, is developed, and its theoretical justification is provided. Finally, we consider a backward problem with non-smooth initial status. We develop an accurate and stable inversion scheme by leveraging multi-temporal observations through data assimilation. For all these inverse problems, numerical experiments are presented to show the effectiveness and efficiency of the proposed methods.



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A theory of computational resolution limit for GMMs

Hai Zhang (haizhang@ust.hk)

Hong Kong University of Science and Technology

Abstract: We study the model selection and parameter estimation problem for 1D Gaussian mixture models, where one aims to estimate the number of Gaussian components and mixing distribution from independent and identically distributed (i.i.d.) samples generated by the model. We introduce the notion of "Computational Resolution Limit", which provides a rigorous framework for determining when a simpler model can effectively approximate a more complex one without significant loss of information. In the context of GMMs, this approach establishes a lower bound on the sample complexity required for accurate model order selection. We further propose a Fourier-based approach to estimate both the model order and the mixing distribution. Our algorithm utilizes Fourier measurements constructed from the samples, and our analysis demonstrates that its sample complexity matches the established lower bound by the Computational Resolution Limit, thereby confirming its optimality.

Internal layer solutions and coefficient recovery in time-periodic reaction-diffusion-advection equations

Ye Zhang (ye.zhang@smbu.edu.cn)

Moscow-BIT Shenzhen

Abstract: In this talk, I will investigate the non-stationary reaction-diffusion-advection equation, emphasizing solutions with internal layers and the associated inverse problems. We examine a nonlinear singularly perturbed partial differential equation (PDE) within a bounded spatial domain and an infinite temporal domain, subject to periodic temporal boundary conditions. A periodic asymptotic solution featuring an inner transition layer is proposed, advancing the mathematical modeling of reaction-diffusion-advection dynamics. Building on this asymptotic analysis, we developed a simple yet effective numerical algorithm to address ill-posed nonlinear inverse problems aimed at reconstructing coefficient functions that depend solely on spatial or temporal variables. Conditions ensuring the existence and uniqueness of solutions for both forward and inverse problems are established. The proposed method's effectiveness is validated through numerical experiments, demonstrating high accuracy in reconstructing coefficient functions under varying noise conditions.

Inverse problems for nonlinear fractional magnetic Schrodinger equations

Ting Zhou (ting_zhou@zju.edu.cn)

Zhejiang University

Abstract: This talk focus on the forward problem and inverse problem for the fractional magnetic Schrodinger equation with nonlinear electric potential. We first investigate the maximum principle for the linearized equation and apply it to show that the problem is well-posed under suitable assumptions on the exterior data. Moreover, we explore uniqueness of recovery of both magnetic and electric potentials.

Error analysis for the numerical reconstruction of conductivity in elliptic equations

Zhi Zhou (zhi.zhou@polyu.edu.hk)

The Hong Kong Polytechnic University

Abstract: Parameter identification for partial differential equations constitutes a broad class of inverse problems. Traditionally, these problems are addressed through optimization approaches, which are then discretized for practical numerical implementation using finite difference, finite element, or neural network approximations. A key challenge in this context is deriving a priori error estimates for the numerical reconstruction of the target parameter. In this talk, we present our recent work on establishing convergence rates for finite element methods in recovering a diffusion coefficient in an elliptic equation. This is achieved by carefully exploiting relevant stability results. Moreover, the approach can be extended to unsupervised learning methods using fully connected neural networks, as well as to multi-parameter identification problems with applications in hybrid physics imaging.