Algebra/Number Theory

Topic A1: Representation theory of finite groups
Proposed by: Prof. Chi-Wai Leung
Description: Representation theory plays an important role in many different areas of modern mathematics such as algebra, number theory, geometry, etc. The aim of this project is to give a starting point of this topic. In this project, we are mainly going to study Jean-Pierre Serre's classical book *Linear Representations of Finite Groups*, Springer, (1977).
Prerequisites: Linear algebra and elementary abstract algebra.

Topic A2: Topics in prime numbers
Proposed by: Dr. Charles Li
Description: The theory of prime numbers is one of the central topics in number theory. It is also one of the oldest topics in mathematics. The first systematic study of prime numbers can be traced back to Euclid 2000 years ago. There are many unsolved problems in the theory of prime numbers, for example, the distribution of primes, infinitude of primes in special forms, Goldbach conjecture, and many more. In the process of solving these problems, many new technique are introduced and bring new insight to mathematics. That includes analytic number theory, Riemann zeta function, sieve method, etc. Prime numbers are also important in modern day cryptography and coding theory. We will cover the following topics: infinitude of primes, distribution of primes, prime number theorem, basic introduction to zeta functions, infinitude of primes in arithmetic progressions.
Prerequisites: Calculus and elementary number theory

Topic A3: An introduction to algebraic geometry and linear algebraic groups
Proposed by: Dr. Charles Li
Description: Algebraic geometry is the study of zeros of algebraic equations. It plays an important role in modern mathematics and has connection to complex analysis, number theory, geometry. Textbooks on modern algebraic geometry, however, is notorious for its abstraction and lack of motivations. We will use linear algebraic groups, which can be described as zeros of algebraic equations with group structures, as the motivation for studying algebraic geometry. Classifications of linear algebraic groups is a beautiful subject on its own. Linear algebraic groups also have important connections to group theory and number theory.
Prerequisites: Elementary abstract algebra

Topic A4: Elliptic curve cryptography
Proposed by: Dr. Kelvin Chun Lung Liu
Description: Algebraic geometry is a branch of mathematics, classically studying zeros of multivariate polynomials. The fundamental objects of study in algebraic geometry are algebraic varieties, which are geometric manifestations of solutions of systems of polynomial equations. One of the most studied algebraic varieties are elliptic curves. The abelian group structures of the points of elliptic curves over finite fields provide the foundation of Elliptic curve cryptography (ECC), which allows smaller keys compared to traditional schemes in cryptography while maintaining an equivalent level of security.
Prerequisites: Elementary abstract algebra

Topic A5: Hypergeometric equations
Proposed by: Prof. Michael McBreen
Description: Hypergeometric equations are a remarkable class of differential equations which have studied since the nineteenth century, yet continue to find surprising applications in modern mathematics. They play important roles in physics, number theory and representation theory, and can be studied using algebraic techniques.
Prerequisites: Linear algebra and multivariable calculus.

Topic A6: Higher composition laws
Proposed by: Prof. Jiu-Kang Yu
Description: The first composition law, that of integral binary quadratic forms, was invented by Gauss as a teenager and had a lot of interesting applications. But for nearly 300 years it was a theory of its own kind without peers, hence sometimes regarded isolated. This changed completely around 2000, when Manjul Bhargava, in his PhD thesis, showed that Gauss's theory is one of a family of composition laws, each of them is fascinating in its own way. These have lots of interesting applications with various number theoretical problems. M. Bhargava received the Fields Medal for this development. One needs little more than what teenager Gauss knew to start learning the theory and peek into some of the modern developments.
Prerequisites: Linear algebra and elementary number theory

Topic A7: Reflection groups
Proposed by: Prof. Jiu-Kang Yu
Description: The theory of finite reflection groups is a very beautiful subject arising from the simple geometric question of generalizing the classification of regular polyhedra (the 5 platonic solids) to higher dimension. The theory of such groups turns out to be surprisingly useful in a lot of seemingly unrelated subjects (Lie groups, singularities, etc). The studying of generalized reflection groups is still a developing subject related broadly to number theory and representation theory.
Prerequisites: Linear algebra and abstract algebra.

Analysis

Topic B1: Topics in ergodic theory and applications
Proposed by: Prof. Dejun Feng
Description: Ergodic theory is a branch of mathematics that studies the behavior of time averages of various functions along trajectories of dynamical systems. It has a wide impact in statistical physics, number theory, probability theory, functional analysis, and other fields. One prominent example is the Green–Tao theorem on arithmetic progressions in prime numbers. Prerequisites: Elementary analysis and measure theory.

Topic B2: Geometry of fractals
Proposed by: Prof. Dejun Feng
Description: Fractals are some irregular and complex objects that often possess self-similarity, infinite complexity, and fractional dimension. They arise everywhere in nature, biological science and human society: coastlines, distributions of galaxies, microscopic surface of materials. Fractals also arise naturally in many branches of mathematics, such as dynamical systems, geometry, number theory and probability theory. The project aims to study various geometric properties of fractal sets and measures.
Prerequisites: Linear algebra and multivariable calculus.

Topic B3: Geometric problems in harmonic analysis
Proposed by: Prof. Po-Lam Yung
Description: In the past decade, insights into various geometric problems have played a key role in many breakthroughs in harmonic analysis, which in turn impacted various fields such as partial differential equations and analytic number theory. Some of these geometric problems can be understood if one has a solid background in linear algebra and multivariable calculus. But the more you know, the more fun it can be: in particular, the project can be taken to the next level, if you happen to know some complex analysis, measure theory, partial differential equations, algebra (rings, fields), number theory and/or topology (differential or algebraic).
Prerequisites: Linear algebra and multivariable calculus.

Computational and Applied Mathematics

Topic C1: Literature Review on Mathematical Models of Rogue Waves
Proposed by: Dr. Hiu-ning Chan
Description: The study of rogue waves has recently attracted researchers in the field of fluid mechanics and oceanography. The occurrences of such unexpectedly large waves in the ocean can cause severe casualties and damages. Mathematically, rogue waves can be modelled by rational functions that are localized in both space and time. For example, the Peregrine breather serves as a classical model of rogue waves in the deep water regime. In this project, we will conduct a literature review on rational solutions of various nonlinear systems. In particular, physical properties such as maximum wave height can be computed with basic knowledge in multivariable calculus. If time permits, numerical approaches to the problem can also be studied.
Prerequisites: Multivariable calculus, differential equations and computer programming.

Topic C2: Interplay of numerical analysis and deep learning in modern computational mathematics
Proposed by: Prof. Eric Chung
Description: Numerical analysis is a classical branch of mathematics. The subject involves the development and mathematical analysis of computational techniques for the approximation of complicated mathematical problems. Recently, data-driven scientific computing has gained much attention, and provides a complementary technique to strengthen the ability of classical approaches to solve mathematical problems. The success is based on the vast amount of available data and computational deep learning techniques.
Prerequisites: Linear algebra, multivariable calculus and computer programming.

Topic C3: Prediction of sleep apnoea in child by analysing 3d facial structures
Proposed by: Prof. Ronald Lui
Description: Sleep apnoea (SA) is a sleep disorder where a person has pauses in breathing or periods of shallow breathing during sleep. The disease is especially common in small children, which can be fatal if they are not given appropriate treatment. Recently, it is believed that SA can be detected from the facial structure. In this project, our goal is to develop a tool for the prediction of SA in children. We will develop a smartphone app, which can construct a 3D human face by taking several photos of the patient from different angles. The 3D face will then be analyzed using mathematical models, such as geometry. Students will learn some basic mathematical techniques related to the project, as well as implement the proposed algorithms using basic programming language such as MATLAB. The research project will be conducted in English.
Prerequisites: Linear algebra, multivariable calculus and computer programming.

Topic C4: Early detection of Alzheimer’s disease using shape analysis of brain subcortical structures
Proposed by: Prof. Ronald Lui
Description: Early Detection of Alzheimer’s Disease Using Shape Analysis of Brain Subcortical Structures Alzheimer’s disease (AD) is neurodegenerative disease that usually starts slowly and gradually worsens over time. It is affecting millions of people worldwide

every year. The cause of the disease is poorly understood. Early detection of the disease is important for providing timely and suitable treatment to patients. Recently, it is believed that the shapes of brain subcortical structures are important indictors for the AD diagnosis. In this project, our goal is to develop a tool for the shape analysis of brain subcortical structures using mathematical techniques such as geometry. Students will learn some basic mathematical techniques related to the shape analysis of subcortical structures, as well as implement the proposed algorithms using basic programming language such as MATLAB. The research project will be conducted in English.
Prerequisites: Linear algebra, multivariable calculus and computer programming.

Topic C5: Worked examples on mathematical methods in physical science
Proposed by: Dr. Jeff Chak-Fu Wong
Description: In this research project, we aim to include four different topics of mathematical methods in physical science: potential theory, Newtonian mechanics, special functions and integral equations. Your tasks will be to deliver/provide:

- an overview of the current development of each above mentioned topic in layman's terms,
- a set of worked examples through theoretical, computational and high level computing programming (animation and graphical visualization) aspects of analyzing these problems, and
- a user-friendly interface platform for promoting the importance of mathematical physics and their related problems and to attract a large number of users from inside and outside the campus to enhance their awareness of the applications of mathematics.

Procedure: Each participant will work on each topic. The length of the project is four/five months. We will meet every week and review your progress. Four researchers are needed.
Prerequisites: Linear algebra, multivariable calculus and computer programming.

Topic C6: Artificial neural network based solution of opinion dynamics
Proposed by: Dr. Jeff Chak-Fu Wong
Description: In this research project, we focus on the numerical solutions of opinion dynamics in a group-based population with a heterogeneous bounded interval using artificial neural network techniques. Your tasks will be to deliver/provide:

- an overview of the current development of opinion dynamics in terms of three aspects: open-minded, moderate-minded and closed-minded agents,
- a set of experiments for analyzing opinion dynamics problems together with the artificial neural network using techniques incorporating theoretical, computational, and high-level computing programming (animation and graphical visualization), and
- a user-friendly interface platform for promoting the importance of opinion dynamics and their related problems and to attract a large number of users from inside and outside the campus to enhance their awareness of the applications of mathematics.

Procedure: The length of the project is four/five months. We will meet every week and review your progress. Students who like challenge should apply.
Prerequisites: Linear algebra, multivariable calculus, differential equations and computer programming.

Topic C7: Real image super-resolution
Proposed by: Prof. Tieyong Zeng
Description: Deep learning-based image super-resolution has achieved great improvement in recent years. However, these models are often accompanied by a complex network structure and a large number of parameters, which makes the model consume a lot of storage space, memory, and execution time during training and use. This is not conducive to the construction of real-time and efficient application systems. At the same time, existing models are often only suitable for simulating down-sampled low-resolution images and it is difficult to obtain good results on real images. Therefore, a flexible and efficient super-resolution reconstruction framework for real images is essential. In this research, we will design a lightweight and efficient feature extraction module, build a prior knowledge guided SR model, and propose a new loss function for real image reconstruction.
Prerequisites: Linear algebra, multivariable calculus and computer programming.

Topic C8: General image denoising network design
Proposed by: Prof. Tieyong Zeng
Description: Image denoising is a challenging inverse problem due to complex scenes and information loss. Recently, various methods have been considered to solve this problem by building a well-designed convolutional neural network (CNN) or introducing some hand-designed image priors. However, none of these problems can solve the basic problem in the image denoising task: a model can suit for different noise level and different noise type. Existing methods usually train specialized models for different noise levels/types, which greatly limits their applications. By studying the direct potential relationship between different noise levels/types, we aim to explore a general image denoising network. In order to achieve it, we will introduce self-supervised learning and meta-learning strategies. Meanwhile, a lightweight and efficient CNN model will be designed for the backbone.
Prerequisites: Linear algebra, multivariable calculus and computer programming.

Geometry/Topology

Topic D1: Hyperbolic geometry
Proposed by: Prof. Thomas Au
Description: This subject is related to Riemann surfaces, differential geometry, topology, knot theory, mathematical physics, geometric groups, and so on. It also builds a prelude to algebraic geometry and other geometries.
Prerequisites: Linear Algebra, multivariable calculus, complex variables, and elementary topology.

Topic D2: Riemann surfaces
Proposed by: Prof. Kwok-wai Chan
Description: The subject of Riemann surfaces lies at the intersection of many different subjects. They play important roles in algebraic geometry, differential geometry, topology, complex analysis, number theory and many others. This is a very rich subject which involves the interplay of techniques of very different favours, and can be approached from various perspectives. Riemann surfaces have also found numerous applications in other subjects such as physics as well as real-life problems.
Prerequisites: Linear Algebra, multivariable calculus, complex variables, and elementary topology.

Topic D3: Lie theory and its geometry
Proposed by: Prof. Kwok-wai Chan
Description: The commutator of two square matrices of the same size gives the simplest example of a structure called Lie bracket. This structure actually appears everywhere in mathematics and provides the bridge for linking different subjects. We will start from scratch and try to explore how such Lie structures arise in geometry.
Prerequisites: Linear algebra and multivariable calculus

Topic D4: Isoperimetric problems
Proposed by: Prof. Martin Li
Description: The classical isoperimetric problem asks for the shape of the closed plane curve with a given length which encloses the maximum area. The solution to the isoperimetric problem is given by a circle and was known already in Ancient Greece. The isoperimetric problem has been extended in multiple ways, for example, to curves on surfaces and to regions in higher dimensional spaces. These isoperimetric problems form an active field of research in several areas of mathematics, including differential geometry, discrete and convex geometry, probability, Banach spaces theory and partial differential equations etc. Isoperimetric problems are also widely applicable in other subjects like theoretical physics, and capillarity problems.
Prerequisites: Multivariable calculus

Topic D5: Geometric flows of curves
Proposed by: Prof. Martin Li
Description: Geometric flows are evolution equations at the intersection of differential equations and geometry. Typically one deforms geometric objects by geometric quantities, like

curvatures, and the equations that arise are non-linear parabolic equations These equations have extensive applications to physical and geometric problems arising in industry, material science, computer vision and image processing, physics, and pure mathematics. The geometric heat equation for one dimensional curve is called curve shortening flow, which exhibit a number of techniques in the field of geometric flows but in an elegant and less technical way.
Prerequisites: Multivariable calculus and differential equations

Topic D6: Geometry of classical mechanics
Proposed by: Prof. Luen-Fai Tam
Description: There are two major mathematical formulations of classical mechanics known as Lagrangian mechanics and Hamiltonian mechanics. These approaches are closely related to many other branches of geometry and mathematical physics such as minimal surfaces, symplectic geometry and general relativity.
Prerequisites: Linear algebra and multivariable calculus.

Topic D7: Knot theory and quantum topology
Proposed by: Prof. Zhongtao Wu
Description: A knot is simply an embedded circle in the Euclidean 3-space. But the theory of knots, which has a long history dating all the way back to the 18^{th} century, is surprising rich. It is not only important in topology and geometry, but also in quantum physics as well as DNA research.
Prerequisites: Linear algebra, multivariable calculus and elementary topology.

Topic D8: Morse theory and symplectic topology
Proposed by: Prof. Zhongtao Wu
Description: Morse theory allows us to analyse the topology of a space (or manifold) by studying differentiable functions on that space. It has a wide range of applications in geometry and topology. A far-reaching generalization of Morse theory gives rise to Floer theory, which plays an indispensable role in modern day symplectic topology
Prerequisites: Linear algebra, multivariable calculus and differential geometry.

Partial Differential Equations

Topic E1: Some mathematical aspects in kinetic theory
Proposed by: Prof. Renjun Duan
Description: The Boltzmann equation is the basis for the kinetic theory of gases and provides a useful tool for the mathematical study of nonequilibrium phenomena in statistical mechanics. A fundamental aspect in kinetic theory is to understand the limiting procedure related to either the validity of the Boltzmann equation from the large number Newtonian particle system or the approximation of the Boltzmann equation through the fluid dynamical system. Meanwhile, the Boltzmann equation itself is of the very importance for describing the motion of a rarefied gas flow in a domain with boundaries. The existence and stability issues on various fundamental physical problems, such as the Couette flow, Poiseuille flow and thermal transpiration, are far from being understood in a rigorous sense in kinetic theory. The research on this topic provides an opportunity to explore the elementary mathematical background of kinetic theory.
Prerequisites: Linear algebra and multivariable calculus.

Topic E2: Geometric wave equations
Proposed by: Prof. Chenyun Luo
Description: This project focuses on understanding the geometric tools that arise from the study of the hyperbolic partial differential equations. We shall, in particular, investigate the linear and semi-linear wave equations from a geometric point of view. Topics include classical energy estimates, Lorentzian vector fields, and the null frame. The geometric wave equations serve as the foundation for more advanced geometric hyperbolic equations, such as Einstein's equation of the general relativity.
Prerequisites: Linear algebra and multivariable calculus.

Topic E3: Navier-Stokes system and related problems
Proposed by: Prof. Zhouping Xin
Description: The well-posedness theory of D-dimensional incompressible Navier-Stokes Equations is one of the Seven Millennium problems posed by the Clay Institute. Around this famous problem, there are many interesting problems which can be studied for understanding the challenges and difficulties from both theory of PDEs and fluid dynamics, such as existence theory of the Leray-Hopf weak solutions, uniqueness and non-uniqueness of weak solutions, possible finite time blow up of solutions, qualitative behave of solutions.
Prerequisites: Linear algebra and multivariable calculus.

Topic E4: Shock wave theory and Burger's equation
Proposed by: Prof. Zhouping Xin
Description: Shock wave theory is the central topics for nonlinear hyperbolic equations. To understand the complicated theory for complex system, a deeper understand can be achieved by studying the pro-type example: Burger's Equation. This will include formation of shock waves in finite time, loss of uniqueness, entropy conditions, vanishing viscosity methods, Hopf-cole Transformation, L1-contraction, Riemann problems, nonlinear hyperbolic waves, behaviour of entropy solutions.
Prerequisites: Linear algebra and multivariable calculus.

Topic E5: First-Order Partial Differential Equations and Its Role in Physics and Geometry
Proposed by: Prof. Yong Yu
Description: First-order partial differential equations play key roles in both mathematics and physics. Their solutions have deep connections with the topology of the spatial domain. This project is devoted to introducing various first-order partial differential equations in 3D. The equations include Euler equation for fluid, Helmhotz equation, Maxwell equation and equation for Z/2 harmonic spinors. Recently several breakthroughs have been made in searching solutions to these equations with knotted and linked structures. We plan to use this project to introduce stories behind.
Prerequisites: Linear algebra and multivariable calculus.