2022-2023 UROP Projects

1. **Supervisor: Prof. Martin LI**
   Geometric problems in mathematical relativity

   Einstein's theory of general relativity is one of the major scientific triumphs in the twentieth century which can be used, for example, to yield precise location in our GPS and to explain the formation of black holes in our universe. The theory is best described mathematically by differential geometry and partial differential equations. It turns out that many of the problems in relativity are geometric in nature and thus it is natural to apply the methods of analysis and differential equations to address these questions. In this project, we will start from the basic concepts in geometry like Riemannian/Lorentzian metrics, covariant derivative, tensor calculus and the various notion of curvatures. After that, we will study the mathematical framework for both special and general relativity and discuss several relativistic concepts like the Cauchy problem for Einstein equations, stress-energy tensor, Schwarzschild spacetime, ADM mass and geodesic incompleteness. Prerequisites: a solid foundation in multivariable calculus and linear algebra, some prior exposure to differential geometry and partial differential equations will be useful but not absolutely necessary.

2. **Supervisor: Prof. Zhongtao WU**
   Knot theory and quantum topology

   A knot is simply an embedded circle in the Euclidean 3-space. But the theory of knots, which has a long history dating all the way back to the 18th century, is surprising rich. It is not only important in topology and geometry, but also in quantum physics as well as DNA research.
   Prerequisites: Linear algebra, multivariable calculus and elementary topology.

3. **Supervisor: Prof. Renjun Duan**
   Kinetic Theory and Its Applications

   Kinetic theory provides a useful mathematical tool for the study of equilibrium or nonequilibrium systems in statistical mechanics. The task is to look for the velocity distribution function of gas particles via which the macroscopic fluid properties of gas can be further understood. A fundamental aspect in kinetic theory, related to the Hilbert's 6th problem, is to justify the asymptotic limit either from the reversible Newtonian particle system to irreversible kinetic equations or from kinetic equations to the fluid dynamic systems such as the Euler or Navier-Stokes equations in classical mechanics. Notable applications of kinetic theory include the study of electron transport in plasmas, neutron transport in nuclear reactors, phonon transport in superfluids, transfer in planetary and stellar atmospheres, and many others. This research project would provide interested students an opportunity to touch some advanced topics on kinetic theory.
   Prerequisites: Multivariable calculus.

4. **Supervisor: Prof. Chan Kwok Wai**
   Complex Morse Theory

   We will investigate complex Morse theory, also called Picard-Lefschetz theory, in this project. This theory studies the topology of a complex manifold by looking at the critical points of a holomorphic function on the manifold as well as the corresponding vanishing cycles and monodromy data. It is a holomorphic analogue of real Morse theory. This theory has very useful applications to the topology of complex projective varieties and symplectic manifolds.
   Prerequisites: basic manifold topology and complex geometry

5. **Supervisor: Dr. Charles LI**
   Number Theory in the spirit of Liouville

   Liouville is one of the greatest mathematicians in the 19th century. He introduced a powerful, yet elementary method into number theory. Liouville's idea is to introduce some elementary (but not easy to prove) identities. The identities can be used to provide elementary proofs of many profound number theoretical results such as Fermat's two square sum theorem, Lagrange's four-square theorem, Gauss' theorem on three triangular numbers, among other things. We will follow the book {\it Number Theory in the Spirit of Liouville by Kenneth William}. One main goal is to type up the solutions of the exercises in the book.
   (The book is available through CUHK Library:
   https://julac-cuhk.primo.exlibrisgroup.com/permalink/852JULAC_CUHK/16slfhk/alma991039543224803407)

   Prerequisites: Number theory.

6. **Supervisor: Prof. Michael McGreen**
   Cap set problem
   The cap set problems asks about how large can a subset of $\mathbb{Z}/p\mathbb{Z}^n$ be that contains no arithmetic progression. The Cap Set Conjecture says that the largest subset of $\mathbb{Z}/3\mathbb{Z}^n$ which contains no arithmetic progress, has size at most c^n with $c < 3$. In 2017, Ellenberg and Gijswijt solved the problem. A detailed proof can be found in https://lean-forward.github.io/e-g/CapSets.pdf
   In this project, we will study the proof, generalizations, and applications.
   Prerequisites: Linear algebra, finite field, algebra.

7. **Supervisor: Prof. Man-Chun LEE**
   Comparison geometry in Riemannian geometry
   In this project, we wiloverview the comparison theory in studying manifolds with curvature bounded from below and study their extension on singular spaces.

8. **Supervisor: Prof. Michael Mcbreen**
   Tropical Geometry
   Tropical geometry is a recent and powerful technique for analysing geometric spaces using combinatorial methods, by studying the limits of these spaces as certain parameters tend to infinity. In this project, you will learn the fundamentals of tropical geometry and apply them to certain problems arising from representation theory and mirror symmetry.

9. **Supervisor: Dr. Ng Ming Ho**
   Introduction to Modular forms and cusp forms

   Modular forms are complex valued-functions on the upper half plane, satisfying certain transformation rules. They contain many interesting number theoretical properties. They are related to quadratic forms, elliptic curves, representation theory and play an important role in proving Fermat's last theorem. In this topic, we will study modular forms and their applications if time permits.
   Prerequisite: Must have learned Complex analysis and topology. Strong background in analysis is recommended

10. **Supervisor: Prof. Dejun Feng**
    Topics in ergodic theory and applications

    Ergodic theory is a branch of mathematics that studies the behavior of time averages of various functions along trajectories of dynamical systems. It has a wide impact in statistical physics, number theory, probability theory, functional analysis, and other fields. One prominent example is the Green–Tao theorem on arithmetic progressions in prime numbers. Prerequisites: Elementary analysis and measure theory.

11. **Supervisor: Dr. Jeff Wong**
    The Mathematics of Game and Gambling

    In this research project, we focus on the numerical solutions of various game and gambling activities using probability, algorithmic game theory and artificial intelligence algorithms. Gambling should not be seen as a way to make money. Gambling is a game of chance, and there's no guarantee that you are going to win. Your task will be to deliver/provide: • an overview of the current development of each above-mentioned topic in layman's terms, • a set of experiments for analyzing game and gambling activities by utilizing probability, algorithmic game theory and artificial intelligence algorithms using techniques incorporating theoretical, computational, and high-level computing programming (animation and graphical visualization), and • a user-friendly interface platform that will promote the importance of game and gambling mathematics and their related problems and attract a large number of users from inside and outside the campus to enhance their awareness of the applications of mathematics. Procedure: The length of the project is five/six months. We will meet every week and review your progress. Students who like a challenge should apply. Prerequisites: Linear algebra, multivariable calculus, probability and computer programming.

12. Supervisor: Prof. Ronald Lui
Medical image & disease analysis using mathematical artificial intelligence

Mathematical artificial intelligence combines mathematical theories with A.I. technologies. With the combination of mathematical tools, the accuracy and efficiency of A.I. techniques can be significantly improved. In the field of medical imaging, the analysis of medical image data can help understand and diagnose diseases. Building mathematical models with machine learning tools is believed to be a promising direction for this purpose. In this project, we will explore various deep learning based mathematical models for various medical image analysis problems. These include solving numerous medical image processing problems and developing diagnostic tools for different diseases, such as Alzheimer's disease and obstructive sleep apnea.

13. Supervisor: Prof. Tieyong Zeng
Deep learning, from model to application

Deep learning, especially in areas such as image processing, has been prosperous in the past 20 years and has promoted the progress of many industries. Now, deep learning models are widely adopted in medical, social, shopping, industrial production, and other scenarios. Therefore, it is meaningful work to explore new application scenarios and improve the current model to adapt to new scenarios. We developed three deep-learning application scenarios involving visual (object) detection, visual navigation, and intelligent diagnosis and treatment. We will mainly focus on 1. Exploring and deploying efficient and lightweight models; 2. Researching and reproducing advanced algorithms in the industry; 3. Algorithm optimization for practical problems, such as autonomous obstacle avoidance in visual navigation. Candidates familiar with Linux, C language, deep learning, and PyTorch are preferred. Of course, the most important thing is to have the ability and attitude of continuous learning.