2024-2025 UROP Projects

1. **Supervisor: Prof. Chenyun LUO**
   Rotational Water Waves

   The motion of water waves is described by Euler equations in a domain with a moving surface boundary. It is well-known that for sufficiently small initial data, the irrotational water waves admit a global-in-time small solution. Recently, in a joint work with Hu and Yao, we showed that the solution of water waves with generic small rotational initial data does not remain small for all time. In this project, we investigate the possible long-time behavior of water waves equipped with certain classes of rotational initial data.

2. **Supervisor: Dr. Ming Ho NG**
   Bertrand's postulate

   In 1845 Bertrand postulated that there is always a prime between n and 2n.
   In 1932, as one of his earliest mathematical publications, Erdos gave a beautiful elementary proof using nothing more than a few easily verified facts about the middle binomial coefficient. In this project is to investigate Erdos's original proof and its variants. We also discuss some problems of combinatorial number theory related to Bertrand's postulate.

   Prerequisites: Complex analysis, Abstract algebra and Number theory
   Additionally, a strong background in analysis would be helpful.

3. **Supervisor: Prof. Kwok Wai CHAN**
   Complex Morse Theory

   We will investigate complex Morse theory, also called Picard-Lefschetz theory, in this project. This theory studies the topology of a complex manifold by looking at the critical points of a holomorphic function on the manifold as well as the corresponding vanishing cycles and monodromy data. It is a holomorphic analogue of real Morse theory. This theory has very useful applications to the topology of complex projective varieties and symplectic manifolds.

   Prerequisites: basic manifold topology and complex geometry

4. **Supervisor: Dr. Kelvin Chun Lung LIU**
   <u>Factorizing large integers</u>
   Factorizing large integers is a fundamental problem in cryptography and number theory with significant implications for security and computation. One of the most notable applications of integer factorization lies in the RSA encryption scheme, which relies on the difficulty of factoring the product of two large prime numbers to ensure the security of encrypted data. To tackle the challenge of factorizing large integers efficiently, various algorithms have been developed, with the Quadratic Sieve and the Number Field Sieve standing out as two prominent methods. The Number Field Sieve is the most powerful algorithm known for factorizing large integers. It operates in the realm of algebraic number theory and utilizes sophisticated mathematical concepts to break down large numbers into smaller factors. The Number Field Sieve has been instrumental in breaking several records for integer factorization and remains a cornerstone in the field of cryptanalysis.

5. **Supervisor: Prof. Gary Pui Tung CHOI**
   <u>Applied and computational geometry</u>

   Geometry plays an important role in many modern science and engineering applications. In this project, students will work on selected problems in applied and computational geometry. Potential topics include the development of fast and accurate geometric mapping algorithms, biological and medical shape analysis using geometric approaches, and the mathematical design of novel engineering materials and structures.

6. **Supervisor: Prof. Eric Tsz Shun CHUNG**
   <u>Scientific Computing</u>

   This project will focus on some modern applied mathematical methods for scientific computing.

7. **Supervisor: Prof. Bangti JIN**
   <u>Scientific Machine Learning Meets Inverse Problems</u>

   We plan to explore using machine learning techniques to solve problems in solving direct and inverse problems for differential equations, e.g., theoretical analysis, algorithmic design, optimization algorithms and efficient training.

8. **Supervisor: Prof. Ronald Lui**
   <u>Medical Diagnosis using Computational Differential Geometry</u>

   This project aims to utilize computational differential geometry techniques, including computational conformal geometry and quasiconformal geometry, for medical image analysis. The goal is to develop tools that assist in the early detection of illnesses, facilitating timely treatment. We will focus on two specific diseases: Adolescent Idiopathic Scoliosis (AIS) and Childhood Obstructive Sleep Apnea (OSA). Geometric models will be developed to analyze medical images for the early diagnosis of these conditions. Collaboration will be established with the medical school at CUHK and hospitals in Hong Kong to ensure the practical applicability and clinical relevance of our work.

9. **Supervisor: Prof. Kuang HUANG**
   Dimension Reduction via Manifold and Deep Learning

   In engineering and data science, observational data are often high-dimensional, in the format of image/text/sound/video, but they can typically be represented in a low-dimensional space. We are interested in methods for dimension reduction, including both traditional manifold learning approaches and modern deep learning techniques. The aim is to reduce high-dimensional data into meaningful low-dimensional representations while preserving data fidelity. Additionally, we are interested in connecting manifold learning and deep learning, from perspectives such as dynamical systems, to analyze and understand the mechanisms underlying these techniques.