2023-2024 UROP Projects

1. **Supervisor: Prof. Xiaolu Tan**
   Numerical method for some parabolic PDEs arising in financial mathematics

   In financial mathematics, the option pricing problem usually leads to a parabolic partial differential equation (PDE). The numerical resolution of such PDEs consists in an important subject in the applications. In this project, we will first review and implement some numerical methods for linear parabolic PDEs. Extensions to the case of the nonlinear PDEs will be considered in a second step.

2. **Supervisor: Prof. Eric Chung**
   Numerical partial differential equations

   The project will focus on both classical and modern techniques in solving differential equations. Examples of topics are finite difference method, finite element method, reduced order method and data-driven method.

3. **Supervisor: Prof. Tieyong Zeng**
   Deep learning, from model to application

   Deep learning, especially in areas such as image processing, has been prosperous in the past 20 years and has promoted the progress of many industries. Now, deep learning models are widely adopted in medical, social, shopping, industrial production, and other scenarios. Therefore, it is meaningful work to explore new application scenarios and improve the current model to adapt to new scenarios. We developed three deep-learning application scenarios involving visual (object) detection, visual navigation, and intelligent diagnosis and treatment. We will mainly focus on 1. Exploring and deploying efficient and lightweight models; 2. Researching and reproducing advanced algorithms in the industry; 3. Algorithm optimization for practical problems, such as autonomous obstacle avoidance in visual navigation.

   Candidates familiar with Linux, C language, deep learning, and PyTorch are preferred. Of course, the most important thing is to have the ability and attitude of continuous learning.

4. **Supervisor: Prof. Zhongtao Wu**
   Problems in Topology

   We will study various topics in topology. Potential areas include knot theory, low-dimensional topology and geometric topology.

5. **Supervisor: Dr. Ming Ho Ng**
   <u>Introduction to the Hardy–Ramanujan–Littlewood circle method</u>

   The circle method was devised by Hardy and Ramanujan in 1918, with an important variant due to Hardy and Littlewood in 1920 known as the Hardy-Littlewood method. The circle method is a beautiful idea for investigating additive problems in analytic number theory. We shall discuss sums of powers of integers, sums of primes, diophantine inequalities and conclude with a description of Roth's celebrated theorem on arithmetic progressions (if time permits).

6. **Supervisor: Dr. Charles Li**
   <u>Modular forms, trace formulas and applications</u>

   Modular forms are complex valued-functions defined on the upper half plane, satisfying specific transformation rules. They possess fascinating number-theoretical properties and are closely connected to various mathematical areas such as algebraic number theory, quadratic forms, elliptic curves, and representation theory. Notably, modular forms play a crucial role in the proof of Fermat's Last Theorem.
   Trace formulas provide a link coefficients of modular forms, Hecke eigenvalues and class numbers.
   This project aims to explore the theory of modular forms and trace formulas, with a particular focus on their applications in computing class numbers and the algebraic properties of the Fourier coefficients of modular forms.
   Prerequisites: A solid understanding of complex analysis and upper-division algebra is essential for this project.
   Additionally, a strong background in analysis is recommended.

7. **Supervisor: Prof. Yong Yu**
   <u>Analysis on complex fluids</u>

   In this project, we concern about various models on complex fluids. They include but not limited to the MHD equation, liquid crystal systems, and viscoelastic system. We will investigate the possible applications of PDE and differential geometry in these physical models. Recently, many experiments and numerical analysis have been conducted on these models. Fantastic phenomena have been observed. Our aim is to theoretically understand the phenomena observed, and look for possible generalizations.

8. **Supervisor: Prof. Renjun Duan**
   Several topics in conservation laws, fluid dynamic equations and kinetic theory

   The analysis of physical models is one of the fundamental concerns in the development of partial differential equations. In the current project we focus on several research topics arranging from conservation laws and fluid dynamic equations to kinetic theory.

   These topics include: 1) Bianchini-Bressan's proof for constructing vanishing viscosity solutions to nonlinear hyperbolic systems;
   2) Glimm's proof for existence of global small BV solutions to general 1D hyperbolic conservation laws;
   3) Chen-Li-Wei-Zhang's proof for studying the transition threshold problem on the Navier-Stokes equations around the 2D Couette flow in a finite channel;
   4) Guo's proof for global existence of small-amplitude solutions to the Boltzmann equation in general bounded domains; and
   5) Golse-Imbert-Mouhot-Vasseur's proof for developing the De Giorgi-Nash-Moser theory to treat regularity on the spatially inhomogeneous Fokker-Planck equations.

   A student interested in this project is supposed to work on only one topic selected from them.

9. **Supervisor: Prof. Martin Li**
   On positive mass theorems in general relativity

   The positive mass conjecture was an important outstanding question in Einstein's theory of general relativity that concerns the nonnegativity of the total mass (or energy) of the entire universe. This conjecture was resolved in the celebrated work of Schoen-Yau and Witten in the early 1980s and it is closely related to the study of scalar curvature in Riemannian geometry. In this project, we will study various formulations of positive mass theorems and the related Penrose inequality. The goal is to understand the statements and geometric/physical meaning of these important theorems and study some of the more recent proofs. Students are expected to have prior exposure to knowledge in Riemannian geometry and elliptic/parabolic partial differential equations. In particular, concepts like tensor calculations and curvatures should be familiar.

10. **Supervisor: Prof. Gary Choi**
    Applied and computational geometry

    Geometry plays an important role in many modern science and engineering applications. In this project, we will work on selected problems in applied and computational geometry, such as the development of fast and accurate geometric

mapping algorithms, applications of geometric methods to biological shape analysis, and geometric design of engineering structures.
Prerequisites: Linear algebra, multivariable calculus, and computer programming.

11. Supervisor: Prof. Michael McBreen
<u>Symmetric Polynomials</u>

Hall polynomials are families of symmetric polynomials which arise in representation theory and mathematical physics. This project will explore their definition and basic properties, using Macdonald's book Hall polynomials. Familiarity with the symmetric group is expected; some knowledge of root systems and lie algebras would also be helpful.

12. Supervisor: Prof. Kwok Wai Chan
<u>Topics in physical mathematics</u>

The term "Physical Mathematics" refers to mathematics motivated by physics (https://www.physics.rutgers.edu/~gmoor/PhysicalMathematicsAndFuture.pdf). In the last few decades, we have been witnessing a tremendous influence of the development of theoretical physics on that of mathematics. This has led to deep and surprising relations between subjects like Lie theory, geometry, topology and even number theory. This project aims at exploring some of such developments in geometry and quantization. Prerequisites: multivariable calculus, linear algebra, basic topology and differential geometry.

13. Supervisor: Prof. Ronald Lui
<u>Geometry meets Real-World Applications</u>

This undergraduate research project aims to explore the practical applications of geometry, specifically focusing on differential geometry, conformal geometry, and quasiconformal geometry. We will investigate how these geometry theories can be utilized in real-world contexts such as medical image analysis and industrial applications. To achieve our goals, we will collaborate with student helpers who will assist in exploring the latest geometric models relevant to our research. We will then develop and implement innovative geometric models for projects in collaboration with companies and medical professionals. By leveraging the power of geometry, we intend to contribute to the development of advanced techniques and models that can address real-world challenges. Our project seeks to bridge the gap between theoretical concepts in geometry and practical applications, ultimately impacting fields such as medicine and industry. Through this research, we aim to uncover valuable insights and create tangible outcomes that demonstrate the potential of geometry in solving real-world problems.

Prerequisites: multivariable calculus, linear algebra, basic differential geometry.

14. Supervisor: Dr. Kelvin Liu
Quantum computing against standard cryptosystems

RSA, Diffie-Hellman and elliptic curve cryptography are three of the most widely adopted asymmetric cryptosystems. Their security largely relies on the difficulty in solving the integer factorization problem (IFP), the discrete logarithm problem (DLP) and its elliptic curve analogue (ECDLP). In 1994, Peter Shor developed a quantum algorithm called Shor's algorithm which can effectively solve IFP, with the aid of a quantum computer. We will study how Shor's algorithm solves IFP and how it can be extended to solve DLP and ECDLP effectively.

15. Supervisor: Prof. Bangti Jin
Scientific Machine Learning

Scientific machine learning is concerned with using various machine learning techniques to solve direct and inverse problems associated with differential equations. In the project we develop new algorithms for various specific problems, provide relevant theoretical analysis of the algorithms and study training dynamics of different formulations.

16. Supervisor: Prof. Chenyun Luo
Lecture notes for hyperbolic PDEs

I have a set of hand-written notes for MATH 5021, Theory of PDE I, and I would like to recruit undergraduate students to type (and probably expand) these notes in Latex. This project helps students fortify their reading and writing skills in mathematics. Prospective candidates should have prior exposure to linear and nonlinear wave equations, and tensor geometry.