THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1540 University Mathematics for Financial Studies 2016-17 Term 1
Coursework 5

Name: ________________________ Student ID: ___________________ Score: _________

Show your work!

1. Let $\vec{a} = \langle 3, 1, 0\rangle$, $\vec{b} = \langle -4, 2, 1\rangle$, $\vec{c} = \langle 0, -2, 5\rangle$. Find

   (a) $(\vec{a} \times \vec{b}) \times \vec{c}$

   (b) $\vec{a} \cdot (\vec{b} \times \vec{c})$

(a)

$$\begin{aligned}
\vec{a} \times \vec{b} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & 0 \\ -4 & 2 & 1 \end{vmatrix} \\
&= \langle 1, -3, 10\rangle \\
\left(\vec{a} \times \vec{b}\right) \times \vec{c} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & 10 \\ 0 & -2 & 5 \end{vmatrix} \\
&= \langle 5, -5, -2\rangle
\end{aligned}$$

(b)

$$\begin{aligned}
\vec{b} \times \vec{c} &= \langle 12, 20, 8\rangle \\
\vec{a} \cdot \left(\vec{b} \times \vec{c}\right) &= (3)(12) + (1)(20) + (0)(8) \\
&= 56
\end{aligned}$$

2. Consider the plane $\mathcal{P}$ in $\mathbb{R}^3$ which contains the points: $P = (3, 4, 5)$, $Q = (-2, -2, -3)$ and $S = (-1, 1, 2)$. Find an equation in x, y, z which describes $\mathcal{P}$.

Solution:

The vectors $\overrightarrow{PQ} = \langle -5, -6, -8\rangle$, $\overrightarrow{QS} = \langle 1, 3, 5\rangle$ are both parallel $\mathcal{P}$, and are non-parallel to each other. Hence, a normal vector of the plane $\mathcal{P}$ is given by:

$$\overrightarrow{n} = PQ \times QS = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & -6 & -8 \\ 1 & 3 & 5 \end{vmatrix} = \langle -6, 17, -9\rangle.$$

Hence, the plane $\mathcal{P}$ can be described by the equation:

$$-6(x-3) + 17(y-4) - 9(z-5) = 0.$$

3. Let $\mathcal{P}$ be the plane in $\mathbb{R}^3$ which is parallel to the vectors $\vec{v} = \langle 1, 2, -1 \rangle$ and $\vec{w} = \langle 0, 3, 5 \rangle$, and contains the point $P = (-4, 0, 7)$. Find an equation of the form $ax + by + cz = d$ which describes $\mathcal{P}$.

Solution:

A normal vector of $\mathcal{P}$ is given by:

$$\overrightarrow{n} = \overrightarrow{v} \times \overrightarrow{w} = \langle 13, -5, 3 \rangle$$

Hence, the plane $\mathcal{P}$ may be described by:

$$13(x + 4) - 5y + 3(z - 7) = 0,$$

or equivalently:

$$13x - 5y + 3z = -31.$$

4. Consider the function $f(x, y) = \ln(xy)\sqrt{y - x}$.

   (i) Find its natural domain $D \subseteq \mathbb{R}^2$;

   (ii) Sketch D in the xy-plane;

   (iii) State whether the region D is open, closed, or neither.

(i) For $f(x, y)$ to be well-defined, we need

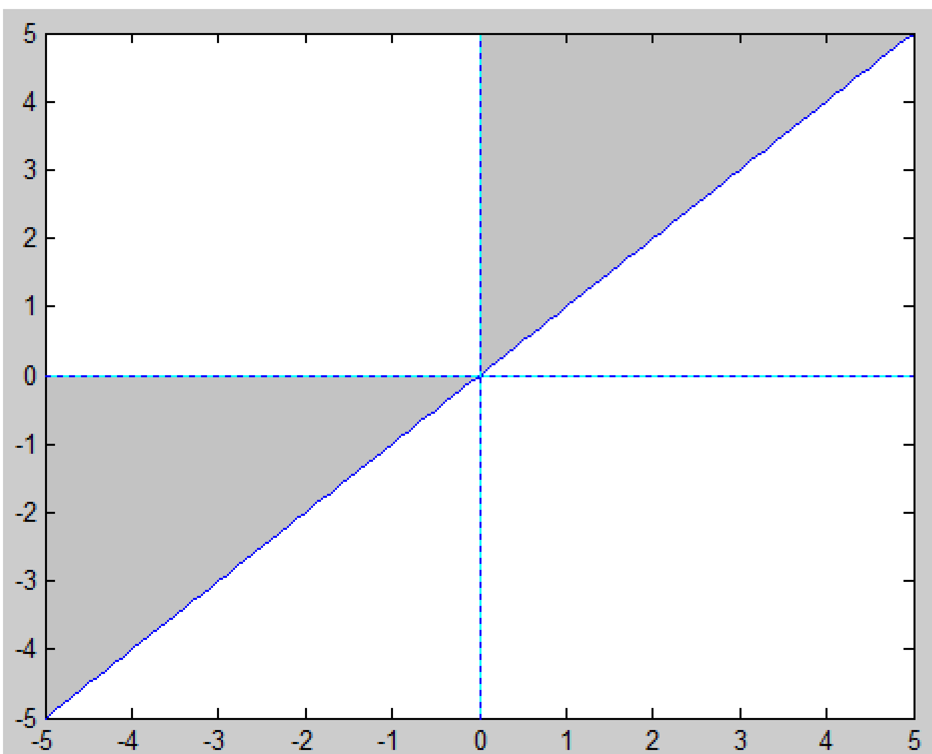
$$xy > 0 \qquad and \qquad y - x \geq 0$$

$$(x > 0, y > 0 \qquad or \qquad x < 0, y < 0) \qquad and \qquad y \geq x$$

Therefore, the natural domain is:

$$D = \{(x, y) : (x > 0, y > 0 \quad or \quad x < 0, y < 0)\, and \quad y \geq x\}.$$

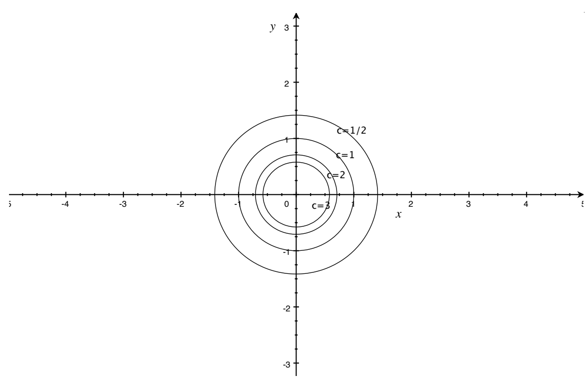
(ii)

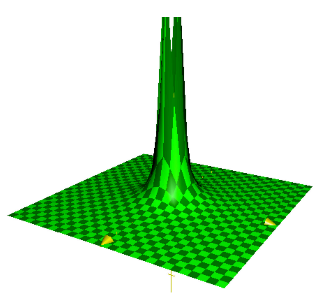


(iii) The domain is neither open nor closed.

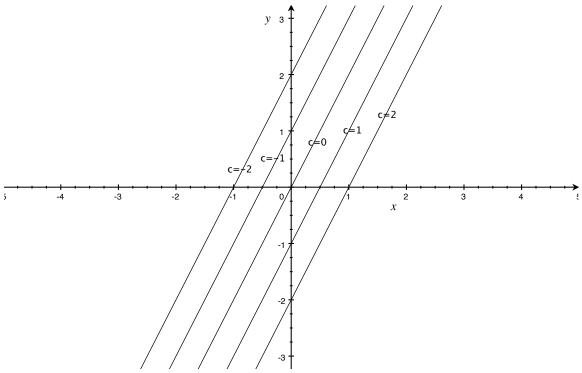
5. Plot the level sets $f(x, y) = c$ of each of the following functions in the xy-plane, then sketch the graph $z = f(x, y)$ of f in the xyz-space.

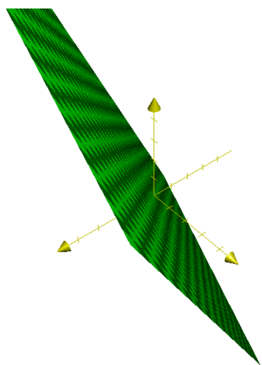
   (a) $f(x, y) = \dfrac{1}{x^2 + y^2}, \quad c = -1, 0, 1/2, 1, 2, 3.$





(b) $f(x,y) = 2x - y\,, \quad c = -2, -1, 0, 1, 2.$





(c) $f(x,y) = \sqrt{1-y^2}\,, \quad c = -1, 0, 1, 2, 3.$