

Revision Exercise for MATH Freshmen 2018-19: Numerical Answers

Section 1.

1. (a) rational number (f) rational number
 (b) rational number (g) irrational number
 (c) rational number (h) irrational number
 (d) irrational number (i) rational number
 (e) rational number (j) irrational number
2. $B = 4, C = 2, D = -1.$
3. (a) $a^2 - 2$ (b) $\sqrt{a^2 - 4}$ (c) $a\sqrt{a^2 - 4}$
4. $A = 1, B = -1, P = 2, Q = 2.$
5. $M = 2, N = 0, P = 1, Q = 2, R = 1.$
6. —
7. —
8. $x = \sqrt{2}$ or $x = -\sqrt{2}$
9. $A = 2, B = 1.$
10. (a) $\frac{7}{4}$ (b) 3 (c) $\frac{49}{25}$ (d) $\frac{13}{37}$
11. 20.
12. (a) $6 : 8 : 15.$ (b) $\frac{9}{10}.$
13. (a) — (b) —
14. —
15. $x = -3.$
16. $x = 64, y = 16.$
17. $(x, y) = (-3, 12)$ or $(x, y) = (\frac{2}{3}, \frac{25}{3})$
18. $(x, y) = (1, -1)$
19. $(x, y) = (1, 1)$ or $(x, y) = (\frac{5}{7}, 5)$
20. $x = 0$ or $x = 2$ or $x = 3.$
21. $\frac{23}{4}$
22. $x = c$ or $x = -c - \frac{7}{2}.$
23. 1, 4.
24. $\frac{-13}{2}, 7$
25. (a) $2x^2 + 7x$ (c) $x^2 + x - 12$
 (b) $2x^2 + 9x - 45$ (d) $5x^2 - 3x - 2$
26. $-2 < \lambda < -1.$
27. (a) $\alpha + \beta = 2 - p, \alpha\beta = p.$ (b) $p = -1$ or $p = 7.$
28. (a) — (b) $k < 1$ or $k > 3.$
29. (a) $pq = (k - 1)^2.$
 (b) $x^2 - (2 - k)x + (k - 1)^2.$
 (c) $0 \leq k \leq \frac{4}{3}.$
30. —
31. (a) $a = -\frac{6}{5}, b = -\frac{24}{5}.$
 (b) $r = 6, s = 4, t = 6$
 (c) $\frac{54}{5}$
32. $x > \frac{4}{5}$
33. $1 < x < 2.$
34. $x \geq 64.$
35. $x < -2$ or $x > \frac{7}{4}$
36. $0 < x < 2.$
37. $x < -4$ or $-1 < x < 1$ or $x > 5.$
38. $-6 < x < -2$
39. $x \leq 2$ or $x \geq \frac{8}{3}.$
40. $-3 < x < 5$
41. $-3 < x < \frac{1}{2}$
42. $x > 1$
43. $a = -2, b = -12.$
44. 60.
45. $\frac{1}{2}(n + 1)(n - 2).$
46. (a) $a = 2, b = 1$ (b) -1140
47. 5
48. $4 : 5.$
49. 1320.
50. (a) $41 + 38i$ (b) $\frac{41}{3125}$ (c) 41
51. (a) Quotient: $3x^2 + 4x + 9.$ Remainder: 17.
 (b) Quotient: $3x^3 - 3x^2 - x - 6.$ Remainder: $-5x + 19.$
 (c) Quotient: $x^4 - x^2 + 1.$ Remainder: $-2.$
52. $a = -7, b = -2.$
53. (a) $(x - 2)^2(2x + 1)$ (b) $(x - 3)(3x - 1)(2x + 1)$

54. (a) 2
(b) —
(c) $x = -1$ or $x = \frac{1}{2}$ or $x = 1$ or $x = -2$
55. $a = -3, b = 2, c = 1$.
56. $f(x) = (a - b)(x - a)(x - b)(x + a + b)$.
57. $A = 4, B = 7, C = 6, D = 3$.
58. —
59. (a) $b = \frac{1}{2a}(f(a) - f(-a)), c = \frac{1}{2}(f(a) + f(-a))$.
(b) $-x + 2$.
60. —
61. 135
62. —
63. (a) — (b) —
64. —
65. (a) — (b) —
16. $\theta = \frac{\pi}{24} + \frac{k\pi}{2}$, or $\theta = \frac{5\pi}{24} + \frac{m\pi}{2}$, or $\theta = \frac{\pi}{2} + n\pi$, where k, m, n are integers.
17. (a) $\sin^2(\mu) - \sin^2(\nu) = \sin(\mu + \nu)\sin(\mu - \nu)$.
(b) $x = \frac{\pi}{10} + K \cdot \frac{2\pi}{5}$ where $K \in \mathbf{Z}$, or $x = M\pi$ where $M \in \mathbf{Z}$.
18. (a) —
(b) Maximum: $-\frac{2}{5}$. Minimum: $-\frac{2}{3}$.
19. $11 : 13 : -7$
20. (a) —
(b) —
21. $p = 1$ or $p = -5$.
22. $1 : 1$.
23. —
24. (a) $\frac{x}{4} + \frac{y}{6} = 1$. (b) —
25. (a) $x + 2y = 0, 7x - y = 0, x - 3y = 0$.
(b) —
(c) $x^2 + y^2 = 25$.

Section 2.

1. 3π square units.
2. $\frac{r^2}{2}(\pi - \sqrt{3})$
3. $a \cos^2(\theta)$
4. $h\left(\frac{1}{\sin(\theta)} + \frac{1}{2\cos(\theta)}\right)$
5. —
6. —
7. —
8. —
9. —
10. —
11. (a) $M = 2, N = 1$. (b) —
12. $-\frac{91}{125}$
13. (a) $\tan(\alpha + \beta) = \frac{4}{7}, \tan(\gamma + \delta) = \frac{3}{11}$.
(b) $\frac{\pi}{4}$
14. —
15. $\theta = \pm\frac{\pi}{2} + 2k\pi$, or $\theta = \pm\frac{\pi}{6} + 2m\pi$, where k, m are integers.
26. 1
27. (a) (3, 4) (b) $\frac{1 \pm \sqrt{129}}{8}$
28. $k = -\frac{11}{2}$ or $k = -\frac{1}{2}$.
29. (a) $4\sqrt{2}$. (b) $a = 32$.
30. (a) $P = \left(\frac{2+r}{1+r}, \frac{2r}{1+r}\right)$.
(b) —
(c) $r = \frac{2}{5}$.
31. (1, 4), (3, 2).
32. (a) Centre: (-4, -6). Radius: 10.
(b) —
(c) $2x + 3y = 0$
(d) $8\sqrt{3}$
33. (a) Radius of C_1 : $\sqrt{2}$. Centre of C_1 : (-1, 2). Radius of C_2 : $3\sqrt{2}$. Centre of C_2 : (3, -2).
(b) $4\sqrt{2}$
(c) —
34. $x^2 + y^2 - 6x - 6y + 9 = 0, x^2 + y^2 - 30x - 30y + 225 = 0$.
35. 5, -5
36. 2
37. (a) Radius: 5. Centre: (5, -4).

(b) $p = 2, q = 8, t = -4.$

(c) i. $x - 2y - 2 = 0$

ii. $(\frac{18}{5}, \frac{4}{5})$

38. (a) $x^2 + y^2 + 2x + 2y - 16 = 0.$

(b) $x + y - 4 = 0.$

(c) $7x - 17y + 68 = 0.$

39. $y^2 = 8x - 16$

40. (a) — (b) $(-1, 0)$

41. (a) $x^2 - y^2 = 4.$

(b) $y = x^2 - x.$

(c) $\frac{x^2}{9} + \frac{y^2}{16} = 1.$

(d) $\frac{x^2}{25} - \frac{y^2}{144} = 1.$

(e) $y^2 = \frac{8 - 4x^2}{(1 - x^2)^2}.$

(f) $x^2 = 1 + \frac{2}{y}.$

42. $\frac{7}{3}$

43. $\mu = 2, \nu = -3.$

44. —

45. (a) $\vec{OB} = 4\mathbf{i} + 3\mathbf{j}, \vec{BQ} = -18\mathbf{i} - 6\mathbf{j}, \vec{OQ} = -14\mathbf{i} - 3\mathbf{j}$

(b) —

46. 49

47. —

48. (a) $|\mathbf{u} + \mathbf{v}| = 2\sqrt{19}.$ (b) $(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) = -20.$

49. $c = 5$ or $c = -1.$

50. (a) $|\mathbf{a}| = 5.$ (b) $\mathbf{a} \cdot \mathbf{b} = 10.$ (c) $m = -\frac{8}{5}.$

51. (a) $5\mathbf{i} + 4\mathbf{j}$ (b) $\frac{1}{\sqrt{5}}$

52. —

53. (a) $\vec{AC} = \mathbf{v} - \mathbf{u}, \vec{BC} = \mathbf{u} + \mathbf{v}.$

(b) 0

(c) —

54. (a) $\vec{AD} = \frac{1}{2}(\mathbf{a} - \mathbf{c}), \vec{BE} = \frac{1}{2}(\mathbf{b} - \mathbf{a}).$

(b) —

55. (a) —

(b) $|\mathbf{m} \times \mathbf{n}| = 6\sqrt{3}.$

Smallest possible area of parallelogram: $4\sqrt{3}.$

Section 3.

1.

$$\frac{1/\sqrt{x+h} - 1/\sqrt{x}}{h}$$

$$= \dots$$

$$= -\frac{1}{(\sqrt{x} + \sqrt{x+h})\sqrt{x} \cdot \sqrt{x+h}}$$

$$\rightarrow -\frac{1}{2x\sqrt{x}} \text{ as } h \rightarrow 0.$$

2.

$$\frac{\tan(x+h) - \tan(x)}{h}$$

$$= \dots$$

$$= \frac{\sin(h)}{h \cos(x+h) \cos(x)}$$

$$\rightarrow \sec^2(x) \text{ as } h \rightarrow 0.$$

3.

$$\frac{(x+h)e^{x+h} - xe^x}{h}$$

$$= \dots$$

$$= e^x e^h + \frac{xe^x(e^h - 1)}{h}$$

$$\rightarrow e^x + xe^x = (x+1)e^x \text{ as } h \rightarrow 0.$$

4. (a) $\frac{1}{4-3x^2}$ (c) $\frac{1}{2\sqrt{1-y^2}}.$

(b) $2\sqrt{(x+1)(x+2)^3}.$ (d) $\frac{1}{\sqrt{1+y^2}}.$

5. $\left. \frac{dy}{dx} \right|_P = -\frac{1}{48}.$

6. $P = 10, Q = -2, R = 3.$

7. $A = 1, B = 0.$

8. Tangent: $y = 2.$ Normal: $x = 1.$

9. $p = -2, q = 0, r = 6.$

10. (a) — (b) $c = 5, m = \pm 3.$

11. —

12. $\frac{15}{13}$ cm per second.

13. $\frac{1}{3\pi}$ m per minute.

14. $\sqrt{\frac{13}{2}}.$

15. (a) $2r \sin(2\theta).$ (b) — (c) $\theta = \frac{\pi}{6}.$

16. $\int \sqrt{2x+1} dx = \frac{(2x+1)^{3/2}}{3} + C,$ where C is an arbitrary constant.

17. $\int x\sqrt{x-1} dx = \frac{2(x-1)^{5/2}}{5} + \frac{2(x-1)^{3/2}}{3} + C$, where C is an arbitrary constant.
18. $\int x \ln(x) dx = \frac{x^2 \ln(x)}{2} - \frac{x^2}{4} + C$, where C is an arbitrary constant.
19. $\int \sin^2(3x) dx = \frac{x}{2} - \frac{\sin(6x)}{12} + C$, where C is an arbitrary constant.
20. $\int \tan^2(4x) dx = \frac{\tan(4x)}{4} - x + C$, where C is an arbitrary constant.
21. $\frac{\pi}{3}$
22. $\frac{8}{105}$
23. $\frac{(\ln(2))^2}{2}$
24. (a) ———
- (b) $\int_0^{\frac{\pi}{2}} \frac{\sin^3(x)dx}{\cos(x) + \sin(x)} = \int_0^{\frac{\pi}{2}} \frac{\cos^3(x)dx}{\cos(x) + \sin(x)} = \frac{\pi - 1}{4}$.
25. (a) ——— (b) ———
26. $y = \frac{\sec^3(x)}{3} - \sec(x) + \frac{2}{3}$.
27. 1.
28. (a) $(0, 0), (1, 0)$. (b) $\frac{8}{15}$.
29. (a) $(0, 0), (2, 8)$. (b) 8.
30. $\frac{96\pi}{5}$.