Mini workshop on Fractals and Related Topics

Date: **7 August 2024 (Wednesday)**
Venue: **Room 219, Lady Shaw Building,**
The Chinese University of Hong Kong, Shatin

9:30-10:30
Professor Pablo Shmerkin (University of British Columbia)
Dynamical self-similarity in higher dimensions

11:00-12:00
Professor Alexia Yavicoli (University of British Columbia)
Thickness and a Gap Lemma in $\mathbb{R}^d$

14:00-15:00
Professor Ying Xiong (South China University of Technology)
Bourgain's probabilistic argument on the Erdős similar problem

15:15-16:15
Professor Bing Li (South China University of Technology)
On orbit complexity of dynamical systems: intermediate value property and level set related to a Furstenberg problem

16:30-17:30
Professor Qinglong Zhou (Wuhan University of Technology)
Uniform Diophantine approximation in continued fractions

All are Welcome

Abstract of the talk:

1. **Professor Pablo Shmerkin** (University of British Columbia)
   Dynamical self-similarity in higher dimensions

   Abstract: Dynamically driven self-similar measures are an ergodic-theoretic generalization of (homogeneous) self-similar measures. In a 2019 work I proved a general statement on the L^q dimensions of this class of measures on the real line, under a weak exponential separation condition. This result turns out to have a wide variety of applications, including the resolution of Furstenberg's slicing conjecture. I will discuss a generalization of the framework to arbitrary dimension, and an application to a higher rank Furstenberg slicing problem. Joint work (in progress) with Emilio Corso.

2. **Professor Alexia Yavicoli** (University of British Columbia)
   Thickness and a Gap Lemma in $\mathbb{R}^d$.

   Abstract: A general problem that comes up repeatedly in geometric measure theory, dynamics and analysis is understanding when two or more (fractal) compact sets intersect. In the real line, the classical Gap Lemma of S. Newhouse, based on the notion of thickness, gives an easily checkable condition for two Cantor sets to intersect, but it is strongly based on the order structure of the reals. I will discuss a recent extension of the notion of thickness, and the Gap Lemma, to higher dimensions.

3. **Professor Ying Xiong** (South China University of Technology)
   Bourgain's probabilistic argument on the Erdős similar problem

   Abstract: In early 1970's, Erdős proposed the similar problem: for any infinite set $A \subset \mathbb{R}$, prove that there exists a set $E \subset \mathbb{R}$ of positive Lebesgue measure such that $aA + b \not\subset E$ for any $a, b \in \mathbb{R}$. In 1987, Bourgain proved this for any triple sum of infinite sets by using some probabilistic argument. This talk will introduce some details of Bourgain's argument.

4. **Professor Bing Li** (South China University of Technology)
   On orbit complexity of dynamical systems: intermediate value property and level set related to a Furstenberg problem

   Abstract: For symbolic dynamics with some mild conditions, we solve the lowering topological entropy problem for subsystems and determine the Hausdorff dimension of the level set with given complexity, where the complexity is represented by Hausdorff dimension of orbit closure. These results can be applied to some dynamical systems such as β-transformations, conformal expanding repeller, etc. We also determine the dimension of the Furstenberg level set, which is related to a problem of Furstenberg on the orbits under two multiplicatively independent maps. This is a joint work with Yuanyang Chang and Meng Wu.

5. **Professor Qinglong Zhou** (Wuhan University of Technology)
   Uniform Diophantine approximation in continued fractions

   Abstract: We consider the multifractal properties of the asymptotic and uniform approximation exponent in continued fractions. As a corollary, we determine the Hausdorff dimension of uniform Diophantine set

   $$\{x \in [0, 1) : \forall N \geqslant 1, \exists n \in [1, N], \text{ s.t } |T^n(x) - y| < |I_N(y)|^{\hat{v}}\}$$

   for a class of quadratic irrational numbers y.