Inverse Problems Seminar

On Smale's 9th problem, generalised hardness of approximation and the limits of AI

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Abstract

Alchemists wanted to create gold, Hilbert wanted an algorithm to solve Diophantine equations, researchers want to make deep learning robust in AI without hallucinations, MATLAB wants (but fails) to detect when it provides wrong solutions to linear programs, etc. Why do we fail in so many of these fundamental cases? The reason is typically methodological barriers. The history of science is full of methodological barriers — reasons for why we never succeed in reaching certain goals. In many cases, this is due to barriers in the foundations of mathematics. This talk introduces new such barriers from foundations: the phenomenon of generalised hardness of approximation (GHA). GHA grows out of our work on Smale's 9th problem on the list of mathematical problems for the 21st century. This phenomenon is not a rare issue — but happens on a daily basis in computations (in particular in AI) — and causes modern software such as MATLAB to fail on basic problems, and even certify nonsensical solutions as correct.

GHA is close in spirit to hardness of approximation (HA) in computer science. Assuming P not equal to NP, HA is the phenomenon that one can easily compute an epsilon-approximation to the solution of a discrete computational problem for epsilon > epsilon_0>0, but for epsilon < epsilon_0 it suddenly becomes intractable. HA was discovered decades ago and has been transformative being the subject of several Goedel, Nevanlinna and ACM prizes. GHA is a similar but distinct mathematical phenomenon that requires a new set of mathematical tools in computation rather than computer science (NB: GHA is independent of P vs. NP). The GHA grows out of the Solvability Complexity Index (SCI) hierarchy framework, and has so far been detected in optimisation, inverse problems, deep learning and AI, as well as computer-assisted proofs. We will discuss in particular the connections to AI.

Date: November 8, 2023 (Wednesday)
Time: 5:00pm – 6:00pm (Hong Kong Time)
ZOOM link: https://cuhk.zoom.us/j/98241093146
Meeting ID: 982 4109 3146

All are Welcome