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Seminar

From Horn Conjecture to Danilov-Koshevoy Conjecture

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Abstract: Consider $n \times n$ Hermitian matices A, B, C such that $A+B=C$. Let $\{\lambda_j(A)\}$,$\{\lambda_j(B)\}$, and $\{\lambda_j(C)\}$ be sequences of eigenvalues of A, B, and C counting multiplicity, arranged in decreasing order. In 1962, A. Horn conjectured that the relations of $\{\lambda_j(A)\}$,$\{\lambda_j(B)\}$, and $\{\lambda_j(C)\}$ can be characterized by a set of inequalities defined inductively. This conjecture was proved true by Klyachko and Knutson-Tao in the late 1990s.

The Horn inequalities can also be characterized by honeycombs, a very intricate combinatorical object invented by Knutson and Tao. From a honeycomb, one can associate a discrete concave function to it, called the *hive*. In a 2003 paper, Danilov and Koshevoy conjectured that

$$\phi_{A,B}(i,j) = \max\{\, tr(AP) + tr((A+B)Q) : P, Q \text{ orthogonal projections}, \quad PQ = 0,\ \mathrm{rank}P = i, \mathrm{rank}Q = j\,\}$$

is a discrete concave function on $\Delta(n) = \{\,(i,j) : 0 \le i, j, i+j \le n\,\}$, the triangle of size n with its integer lattice points. This conjecture says that the combinatorics not only reveals the algebraic relationships (i.e. inequalities of eigenvalues as in the Horn Conjecture), but also the geometric relationship of A, B, and $A+B$. In this talk, I will present some partial results on this conjecture. This is a joint work with H. Bercovici.

Date: Thursday, 20 April 2023
Time: 10:30 am – 11:30 am
Venue: Room 222, Lady Shaw Building,
The Chinese University of Hong Kong, Shatin

All are Welcome!