# Satellite Workshop of The 26th International Domain Decomposition Conference

## DECEMBER 5, 2019 (THURSDAY)
ROOM 222,
LADY SHAW BUILDING, CUHK

organized by Department of Mathematics,
The Chinese University of Hong Kong

**Speakers:**

Prof. Eric Chung (*The Chinese University of Hong Kong*)
Prof. Zdeněk Dostál (*VŠB -Technical University of Ostrava*)
Prof. Martin Gander (*University of Geneva*)
Dr. Shihua Gong (*University of Bath*)
Dr. Yaguang Gu (*Hong Kong Baptist University*)
Dr. Phil. Paolo Piersanti (*University of Graz*)
Prof. Marcus Sarkis (*Worcester Polytechnic Institute*)
Prof. Damien Tromeur-Dervout (*Université de Lyon*)

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# Satellite Workshop of

# The 26th International Domain Decomposition Conference

December 5, 2019 (Thursday)

Room 222, Lady Shaw Building, The Chinese University of Hong Kong

| Program | |
|---|---|
| 0900 – 0940 | Zdenek Dostal (*VŠB - Technical University of Ostrava*)<br>Improving Eiciency of Scalable TFETI/BETI Contact Solvers |
| 0940 – 1020 | Marcus Sarkis (*Worcester Polytechnic Institute*)<br>Spectral Additive Schwarz Methods for Hybrid Discontinuous Garlerkin Discretizations |
| 1020 – 1050 | Coffee break |
| 1050 – 1130 | Damien Tromeur-Dervout (*Université de Lyon*)<br>Acceleration of the Convergence of the Asynchronous Restricted Additive Schwarz Method |
| 1130 – 1210 | Paolo Piersanti (*University of Graz*)<br>Justification of Koiter’s Model for an Elliptic Membrane Shell over an Obstacle |
| 1210 – 1430 | Lunch *(by invitation)* |
| 1430 – 1510 | Martin J. Gander (*University of Geneva*)<br>Iterative Methods for Linear Systems over two Centuries |
| 1510 – 1550 | Shihua Gong (*University of Bath*)<br>Parallel Preconditioners for High Order Discretisations of the Heterogeneous Helmholtz Equation |
| 1550 – 1620 | Coffee break |
| 1620 – 1700 | Yaguang Gu (*Hong Kong Baptist University*)<br>Optimized Schwarz Method for Domains with Non-Conforming Heterogeneities |
| 1700 – 1740 | Eric Chung (*The Chinese University of Hong Kong*)<br>Local Multiscale Model Reduction for Flows in Heterogeneous Media |
| 1830 | Dinner *(by invitation)* |

# Title and abstract of talks

## Local Multiscale Model Reduction for Flows in Heterogeneous Media

**Eric Chung**
*The Chinese University of Hong Kong*

### Abstract

We will present recent advances in developing computational multiscale models for flows in heterogeneous media. We will discuss the motivation, the methodology and some numerical results.

## Improving Eciency of Scalable TFETI/BETI Contact Solvers

**Zdenek Dostal**
*VŠB - Technical University of Ostrava*

### Abstract

The development of scalable solvers for contact problems is a challenging task due to a priori unknown contact conditions which make the problem strongly nonlinear. In the first part of the lecture, we specify the challenges and present the tools that can be used to overcome them. We first briefly review the *TFETI/TBETI* (total finite/boundary element tearing and interconnecting) based domain decomposition methodology adapted to the solution of contact problems of elasticity discretized by matching grids. The scalability of the proposed method is based on a combination of classical estimates due to Farhat, Mandel, and Roux with the results on the rate of convergence of some special quadratic programming and QCQP (quadratic programming – quadratic constraints) algorithms. The theory guarantees that an approximate solution with prescribed relative error can be obtained in $O(1)$ matrix–vector multiplications provided the cost of the projector to the "natural coarse grid" does not dominate the computation. The results apply to the multibody frictionless problems, both static and dynamic, and to the problems with a given (Tresca) friction.

In the rest, we present three extensions starting with *mortar discretization* of the contact conditions for non-matching grids on the contact interface. The generalization is non-trivial due to the not obvious conditioning of inequality constraints. Improved numerical results are obtained by means of *adaptive augmentation* that enhances into the augmented Lagrangian algorithm the information about the current estimate of the active set of the solution.

The final improvement is the *re-orthogonalization based preconditioning* and *re-normalization based scaling* which enables to extend the scope of applications from the problems with constant coefficients to those with homogeneous subdomains. Though there is a number of results for the problems with jumping coefficients, they typically use the preconditioners transforming separable constraints into more general ones that cannot be treated by specialized algorithms.

## Iterative Methods for Linear Systems over two Centuries

**Martin J. Gander**
*University of Geneva*

### Abstract

Iterative methods for linear systems were invented for the same reasons as they are used today, namely to reduce computational cost. Gauss states in a letter to his friend Gerling in 1823: "you will in the future hardly eliminate directly, at least not when you have more than two unknowns".

Richardson's paper from 1910 was then very influential, and is a model of a modern numerical analysis paper: modeling, discretization, approximate solution of the discrete problem, and a real application. More general vector extrapolation methods were then introduced and studied in the pioneering work of Bresinzki, and they can be shown to be equivalent to Krylov method.

It was however the work of Stiefel, Hestenes and Lanczos in the early 1950 that sparked the success story of Krylov methods with the invention of the conjugate gradient method, and there are now many Krylov methods to choose from. For general linear systems, they come in two main classes, the ones that minimize the residual in a Krylov space, and the ones that make the residual orthogonal to it.

This will bring us finally to the modern iterative methods for solving partial differential equations, which also come in two main classes: domain decomposition methods and multigrid methods. Domain decomposition methods go back to the alternating Schwarz method invented by Herman Amandus Schwarz in 1869 to close a gap in the proof of Riemann's famous Mapping Theorem. Multigrid goes back to the seminal work by Fedorenko in 1961, with main contributions by Brandt and Hackbusch in the Seventies.

# Parallel Preconditioners for High Order Discretisations of the Heterogeneous Helmholtz Equation

**Shihua Gong**
*University of Bath*

## Abstract

The Helmholtz equation with large wave number $k$ is difficult to solve numerically mainly because of the highly oscillatory nature of its solutions (on a scale of $\frac{1}{k}$) and the indefiniteness of its standard variational formulation (with smaller discrete Inf-Sup constant as $k$ increases). High order finite element methods can provide efficient approximation to the oscillatory solutions. The indefiniteness of the arising linear system may be much more severe while taking account the waves in heterogeneous media since some rays may be trapped in local regions and the energy in these regions is significantly large. We consider the 'Optimised Restricted Additive Schwarz' (ORAS) preconditioners to solve the linear system arising from the high order discretizations of the heterogeneous Helmholtz equation. The action of these preconditioners requires solutions of independent subproblems with absorbing boundary conditions and (possibly) absorption added in the domain. Then the local solutions are glued together using prolongation operators defined by the composition of a nodal interpolation and a partition of unity. Supporting theory for the symmetrized version of these preconditioners in the case of homogeneous Helmholtz problems is given by I. G. Graham E. A. Spence and J. Zou in [Preprint arXiv:1806.03731.] In this talk we extend this theory to heterogeneous cases and figure out the effect of the underlying finite element discretizations to the preconditioners. We also investigate numerically the role of the 'non-trapping' criterion in the performance of the preconditioners.

# Optimized Schwarz Method for Domains with Non-Conforming Heterogeneities

**Yaguang Gu**
*Hong Kong Baptist University*

## Abstract

In many applications, we need to solve the linear elliptic equations which are of the form $-\nabla \cdot (\nu(x)\nabla u) = 0$ where the diffusivity $\nu$ may vary and oscillate over the whole physical domain. Note that it is usually inconvenient to decompose the physical domain aligned with the discontinuities. In practice, a decomposition of the physical domain into strips or $M \times N$ subdomains is usually used, which may lead to subdomains with non-conforming heterogeneities.

In this talk, we focus on the model problem where the diffusivity consists of two constant pieces $\nu^-$ and $\nu^+$ and present an analysis for two-subdomain nonoverlapping decomposition. we will show that the constant Robin parameter is not robust any more for the model due to the convergence factor of $1 - \mathcal{O}((\nu^+/\nu^-)^{-1/2}(H/h)^{-1/2})$, where $H$ is the diameter of each subdomain and $h$ is the mesh parameter. We will also propose a scaled Robin parameter which is proportional to the diffusivity and then derive the convergence factor of $1 - \mathcal{O}((H/h)^{-1/2})$. Numerical examples show that the proposed parameter choice is also robust for more general heterogeneous problems not covered by the theory.

We will also introduce an Optimized Restricted Additive Schwarz Preconditioned Exact Newton (ORASPEN) method for the model problem where $\nu = \nu(x, u, \nabla u)$. Note that the model problem is now nonlinear. We test some examples using overlapping domain decompositions, showing that the ORASPEN has superior convergence behavior than the RASPEN (the one with Dirichlet transmission conditions). Besides, we show that the scaled Robin parameter is also a better choice than the uniform one.

## Justification of Koiter's Model for an Elliptic Membrane Shell over an Obstacle

**Paolo Piersanti**
*University of Graz*

### Abstract

In this talk, I will present some recent results on obstacle problems for elliptic membrane shells. A specific obstacle problem for the three-dimensional model will be defined and a rigorous asymptotic analysis as the thickness approaches zero will be performed. We will finally compare the obtained result with what is obtained by letting the thickness approach zero in the corresponding Koiter's model subjected to an obstacle.

## Spectral Additive Schwarz Methods for Hybrid Discontinuous Garlerkin Discretizations

**Marcus Sarkis**
*Worcester Polytechnic Institute*

### Abstract

Additive Average Schwarz methods were introduced by Bjorstad, Dryja and Vainikko 97' and designed for solving 2D and 3D elliptic problems with discon-tinuous coefficients across subdomains using classical discretizations. Spectral Additive Schwarz Methods is an upgrade version which can solve 2D and 3D el-liptic problems with discontinuous coefficients across elements. These methods use non-overlapping subdomains and the subdomain iteration is via the coarse space. The methods do not require a coarse triangulation and the coarse prob-lem can be seen as inverting a diagonal matrix plus a sparse low-rank matrix as-sociated to few generalized eigenvectors obtained purely inside the subdomains. The condition number estimate of the preconditioned systems is of the same or-der as the classical Additive Schwarz methods with minimum overlap and with a coarse space, that is, O(H/h), independent of coefficients. In this talk we design and analyze this type of preconditioning for some Hybrid Discontinuous Gar-lekin discretizations with heterogeneous coefficients, possibly with high-contrast coefficients and with the use of local generalized eigenvalues problems in order to recover the robust O(H/h) condition number estimate independently of the coefficients.

# Acceleration of the Convergence of the Asynchronous Restricted Additive Schwarz Method

**Damien Tromeur-Dervout**
*Université de Lyon*

## Abstract

This talk focuses on the acceleration of the convergence of the asynchronous Restricted Additive Schwarz (RAS) iterates. Schwarz methods with asynchronous communications are becoming particular interesting with the development of high performance computers with several thousand of cores and with more and more complex hierarchical communication networks. In these context the use of global reduction operation such as the dot products involving in the GMRES acceleration can be the bottleneck for the performance. For asynchronous RAS , some boundary conditions of some artificial interfaces generated by the domain decomposition cannot been updated for some iterates. In these context the Aitken acceleration should not be applicable as it is based on the pure linear convergence of the RAS., i.e. there exists a linear operator $P$ independent of the iteration that links the error with the searched solution at the artificial interfaces of two consecutives RAS iterates. In the context of asynchronous iteration we do not have a priori the pure linear convergence property as some boundary conditions have not been updated (randomly). We develop a mathematical model of the Asynchronous RAS allowing to set the percentage of the number of the randomly chosen local artificial interfaces where boundary conditions are not updated. Then we show how this ratio deteriorates the convergence of the Asynchronous RAS and how some regularization techniques on the traces of the iterative solutions at artificial interfaces allow to accelerate the convergence to the searched solution.