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Short Course

Complex Analysis and Riemann surfaces

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Abstract:

The theory of Riemann surfaces occupies a very special place in mathematics. It is a culmination of much of traditional calculus, making surprising connections with geometry and arithmetic. It is an extremely useful part of mathematics, knowledge of which is needed by specialists in many other fields. It provides a model for a large number of more recent developments in areas including global analysis, algebraic and complex geometry and mathematical physics.

A Riemann surface is a one-dimensional complex manifold. In complex geometry, it is crucial to be able to construct holomorphic (analytic) objects. By Liouville's theorem, any analytic function on a compact Riemann surface is a constant. Therefore, to construct analytic objects on Riemann surfaces, we usually look for a holomorphic line bundle over a compact Riemann surface. Classically, we need Riemann-Roch theorems and vanishing theorems to construct holomorphic sections. In this short course, we will give a Bergman kernel method to construct holomorphic sections on Riemann surfaces. For prerequisites, we will only assume some undergraduate complex analysis. We will also cover some classical topics in Riemann surfaces, such as Elliptic functions and the Uniformisation Theorem.

The plan is as follows.

- (I) Some complex analysis: Cauchy-Riemann equations, subharmonic functions, sub-mean value inequalities.
- (II) Bergman kernels and Tian's theorem on the complex plane.
- (III) Riemann surfaces: Basic definitions.
- (IV) Calculus on Riemann surfaces: differential forms, de-Rham and Dolbeault cohomology groups, Kodaira Laplacian, harmonic functions, Elliptic P.D.Es on compact Riemann surfaces.
- (V) Holomorphic line bundles over a Riemann surface.
- (VI) Bergman kernels and Tian's theorem on compact Riemann surfaces.
- (VII) Some classical topics about Riemann surfaces.

Date:	6, 13, 20, 27 March 2019 (Wednesdays) 3, 10, 17, 24 April 2019 (Wednesdays)
Time:	4:30 p.m. - 6:30 p.m.
Venue:	Room 222, Lady Shaw Building The Chinese University of Hong Kong, Shatin

All are Welcome!