

Department of Mathematics  
The Chinese University of Hong Kong

For Favour of Posting

數學系  
香港中文大學

Phone: (852) 3943 7988 • Fax: (852) 2603 5154 • Email: [dept@math.cuhk.edu.hk](mailto:dept@math.cuhk.edu.hk) (Math. Dept.)  
Room 220, Lady Shaw Building, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong

# Workshop on Harmonic Analysis

June 26 – 30, 2017

Rm. 222, Lady Shaw Building, CUHK

## Program

### June 26, 2017 (Monday)

- 10:30am - 11:30am      **Po-Lam Yung (The Chinese University of Hong Kong)**  
An introduction to the polynomial method
- 2:30pm - 5:00pm      Free Discussion

### June 27, 2017 (Tuesday)

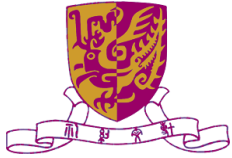
- 10:30am - 11:30am      **Po-Lam Yung (The Chinese University of Hong Kong)**  
The Szemerédi-Trotter theorem
- 2:30pm - 3:30pm      **Joris Roos (University of Bonn)**  
A polynomial Roth theorem on the real line, Part I
- 3:45pm - 4:45pm      **Polona Durcik (University of Bonn)**  
A polynomial Roth theorem on the real line, Part II

### June 28, 2017 (Wednesday)

- 11:30am - 12:30pm      **Po-Lam Yung (The Chinese University of Hong Kong)**  
The joints theorem
- 2:30pm - 5:00pm      Free Discussion

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## Program

### June 29, 2017 (Thursday)

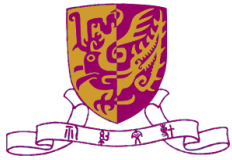
- 10:30am - 11:30am     **Diogo Oliveira e Silva (University of Bonn)**  
Restriction, Keakeya and Decoupling, Part I
- 2:30pm - 3:30pm     **Polona Durcik (University of Bonn)**  
Outer measure  $L^p$  spaces and the bilinear Hilbert transform,  
Part I
- 3:45pm - 4:45pm     **Polona Durcik (University of Bonn)**  
Outer measure  $L^p$  spaces and the bilinear Hilbert transform,  
Part II

### June 30, 2017 (Friday)

- 10:30am - 11:30am     **Diogo Oliveira e Silva (University of Bonn)**  
Restriction, Keakeya and Decoupling, Part II
- 2:30pm - 3:30pm     **Joris Roos (University of Bonn)**  
Outer measure  $L^p$  spaces and the bilinear Hilbert transform,  
Part III
- 3:45pm - 4:45pm     **Joris Roos (University of Bonn)**  
Outer measure  $L^p$  spaces and the bilinear Hilbert transform,  
Part IV

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## Speaker & Title

Speaker: Po-Lam Yung (The Chinese University of Hong Kong)

Title: *An introduction to the polynomial method*

Abstract:

We will introduce the method of polynomial partitioning, a powerful tool that has been introduced in recent years by Dvir, Katz, Guth, Zhang and others in attacking a spectacular range of problems from incidence geometry to harmonic analysis.

Title: *The Szemerédi-Trotter theorem*

Abstract:

Given  $M$  points and  $N$  lines in the plane, what is the maximum number of incidences between them? This is answered by a famous theorem of Szemerédi and Trotter. We will give a proof (following Katz and Tao) using the polynomial method introduced in the previous talk.

Title: *The joints theorem*

Abstract:

Given a finite family of lines in  $\mathbb{R}^n$ , a joint formed by these lines is a point that lies in  $n$  lines from the family, whose directions are linearly independent. How many joints can a family of  $N$  lines form in  $\mathbb{R}^n$ ? We will answer this question using the polynomial method. This is an exposition of work of Carbery, Iliopoulou, Katz, Guth, Quilodran, Kaplan, Sharir, Shustin and Zhang.

Speakers: Polona Durcik (University of Bonn) and Joris Roos (University of Bonn)

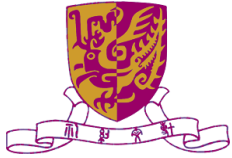
Title: *A polynomial Roth theorem on the real line*

Abstract:

For a polynomial  $P$  of degree greater than one, we show the existence of patterns of the form  $(x, x + t, x + P(t))$  with a gap estimate on  $t$  in positive density subsets of the reals. This is an extension of an earlier result of Bourgain. Our proof is a combination of Bourgain's approach and more recent methods that were originally developed for the study of the bilinear Hilbert transform along curves. This talk is based on a joint work with Shaoming Guo.

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## Speaker & Title

Speaker: Diogo Oliveira e Silva (University of Bonn)

Title: *Restriction, Kakeya and Decoupling*

### Abstract:

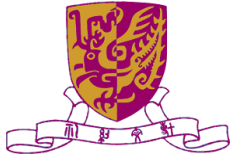
In a recent series of papers, J. Bourgain and C. Demeter provided proofs for the so-called  $\ell^2$  decoupling conjecture in a variety of contexts. In short, contributions of pieces of the Fourier transform localized to pairwise disjoint caps get decoupled via analytic, geometric and topological mechanisms. Tools include wave packet decompositions, parabolic rescaling, and multilinear tools related to the restriction and Kakeya problems from classical harmonic analysis. Consequences are vast, and range from analytic (proof of the discrete restriction conjecture) to combinatorial (improved bounds for additive energies) and number-theoretic (counting solutions to Diophantine equations, progress on the Lindelöf hypothesis). In particular, it is remarkable that significant progress in number theory is achieved via Fourier analysis only, and does not rely on any previous number theoretic results. A number of questions are still open and will likely be the focus of much research in the next few years.

In the lecture on June 29, we focus on the multilinear theory. We establish a square function estimate in the plane which implies the decoupling inequality on a suboptimal range, and discuss a recent short proof by L. Guth of the trilinear Kakeya inequality in 3-dimensional Euclidean space.

In the lecture on June 30, we highlight some elements of the proof of the sharp decoupling theorem for the parabola. We discuss the equivalence of the linear and the bilinear decoupling problems, and set up an induction-on-scales argument that combines the ingredients from the previous lecture.

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## Speaker & Title

Speakers: Polona Durcik (University of Bonn) and Joris Roos (University of Bonn)

Title: *Outer measure  $L^p$  spaces and the bilinear Hilbert transform*

### Abstract:

In this series of four lectures we will present a recent paper by Do and Thiele: " $L^p$  theory for outer measures and two themes of Lennart Carleson united".

The authors of the paper introduce a theory of  $L^p$  spaces based on outer measures. This provides a framework that is well-suited for studying singular integrals in time-frequency analysis such as the bilinear Hilbert transform. In particular, the authors reprove  $L^p$  bounds for the bilinear Hilbert transform, which were first established in groundbreaking work by Lacey and Thiele in the 90s.

First we will introduce the abstract concept of outer measure  $L^p$  spaces and discuss some examples and basic properties.

As a warm-up application of the theory we then reprove  $L^p$  estimates for paraproducts via a version of the Carleson embedding theorem.

Finally, we show how  $L^p$  bounds for the bilinear Hilbert transform can be obtained via the outer measure theory and a generalized Carleson embedding theorem.

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