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Seminar

Group actions, the Mattila integral and continuous sum-product problems

Mr. Bochen Liu
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Date :	May 11, 2017 (Thursday)
Time :	2:00pm – 3:00pm
Venue :	Room 222, Lady Shaw Building, The Chinese University of Hong Kong

All are Welcome

Group actions, the Mattila integral and continuous sum-product problems

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Abstract

The Mattila integral,

$$\mathcal{M}(\mu) = \int \left(\int_{S^{d-1}} |\widehat{\mu}(r\omega)|^2 d\omega \right)^2 r^{d-1} dr,$$

developed by Mattila, is the main tool in the study of the Falconer distance problem. Recently this integral is interpreted by Greenleaf et al. in terms of the L^2-norm of the natural measure on $E - gE$, $g \in O(d)$, the orthogonal group. Following this group-theoretic viewpoint, we develop an analog of the Mattila integral associated with arbitrary groups. As an application, we prove for any $E, F, H \subset \mathbb{R}^2$, $\dim_{\mathcal{H}}(E) + \dim_{\mathcal{H}}(F) + \dim_{\mathcal{H}}(H) > 4$, the set

$$E \cdot (F + H) = \{x \cdot (y + z) : x \in E, y \in F, z \in H\}$$

has positive Lebesgue. In particular, it implies that for any $A \subset \mathbb{R}$,

$$|A(A + A)| > 0$$

whenever $\dim_{\mathcal{H}}(A) > \frac{2}{3}$. We also give a very simple argument to show that on $\mathbb{R}^2$, $\dim_{\mathcal{H}}(E) > 1$ is sufficient for $|E \cdot (E \pm E)| > 0$, where the dimensional threshold is optimal. By taking $E = A \times A$, it follows that

$$|A(A + A) + A(A + A)| > 0$$

whenever $\dim_{\mathcal{H}}(A) > \frac{1}{2}$, which is also sharp. We therefore conjecture that $\frac{1}{2}$ is the best dimensional threshold for $A \subset \mathbb{R}$ to ensure $|A(A + A)| > 0$.