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# Seminar

## ***Stable Recovery of the Factors from a Deep Matrix Product***

*Professor François Malgouyres*  
*Université Paul Sabatier*  
*France*

### Abstract

We study a deep matrix factorization problem. It takes as input the matrix  $X$  obtained by multiplying  $K$  matrices (called factors). Each factor is obtained by applying a fixed linear operator to a short vector of parameters satisfying a model (for instance sparsity, grouped sparsity, non-negativity, constraints defining a convolution network...). We call the problem deep or multi-layer because the number of factors is not limited. In the practical situations we have in mind, we typically have  $K = 10$  or  $20$ . This work aims at identifying sharp conditions on the structures of the problem guaranteeing that we can stably recover the factors using the knowledge of  $X$  and the model for the factors.

We provide necessary and sufficient conditions for the identifiability of the factors (up to a scale rearrangement). We also provide a necessary and sufficient condition named "Deep Null Space Property" (DNSP) (because of the analogy with the usual Null Space Property in the compressed sensing framework) guaranteeing that even an inaccurate optimization algorithm aiming at the factorization stably recovers the factors.

We then provide a second, simpler, version of the DNSP dedicated to models enforcing sparsity in the parameters. To the best of our knowledge, this second criterion is new even when  $K = 1$ , in the compressed sensing context. Again, we prove that this second criterion is necessary and sufficient to guarantee the stable recovery of the factors, even with an inaccurate minimization scheme. In the compressed sensing framework the bounds are analogous to the bounds obtained with the lower-RIP.

We detail the analysis of a practical example where the deep factorization is a convolutional network.

This is a joint work with Joseph Landsberg.

**Date :** January 11, 2017 (Wednesday)  
**Time :** 4:30p.m. – 5:30p.m.  
**Venue :** L5, Science Centre  
The Chinese University of Hong Kong

*All are Welcome*