Homework 2 2024-2025

The Chinese University of Hong Kong
Department of Mathematics
MMAT 5340 Probability and Stochastic Analysis
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Please submit your solutions on blackboard before
11:59 AM, Jan 20th 2025

January 13, 2025

1.

Let X and Y be two random variables with the same discrete uniform distributions, i.e. their probability mass functions are given by

$$P(X = k) = P(Y = k) = \frac{1}{N}, \quad k \in \{1, 2, \ldots, N\}.$$

Suppose that X and Y are independent. Compute $E[X + Y|X]$.

Hint: You may want to recall the elementary formula

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2}.$$

2.

Suppose that X and Y are independent random variables, both with the standard normal distribution, i.e. $X, Y \sim N(0, 1)$. For $\rho \in [-1, 1]$, define

$$Z := \sqrt{1 - \rho^2} X + \rho Y.$$

Show that $E[Z|Y] = \rho Y$.

3.

Fix a probability space $(\Omega, \mathcal{F}, P)$. Let X and Y be two independent continuous random variables with joint density function $\rho_{X,Y}(x, y)$ and the marginal density functions are denoted by $\rho_X(x)$ and $\rho_Y(y)$.

Let $g(X, Y)$ be a function with $E[|g(X, Y)|] < \infty$, and we define

$$f(y) := E[g(X, y)].$$

Show that

$$E[g(X, Y)|Y] = f(Y).$$

Hint: Use Fubini's theorem; also, the proof of Example 1.16 (ii) in the lecture notes may be helpful.

Hint: Recall that for a random variable Z that is $\sigma(Y)$-measurable, there exists some measurable function h such that $Z = h(Y)$. You may use this result without proof.