MMAT5010 Linear Analysis (2024-25): Homework 7
Deadline: 29 Mar 2025

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard.

1. Let X and Y be non-zero normed spaces. Let $B(X,Y)$ be the space of all bounded linear maps from X to Y.
   (a) Show that for any non-zero element $x_0 \in X$, there is $f \in X^*$ such that $f(x_0) = 1$.
   (b) Show that for any pair of elements $x_0 \in X$ with $x_0 \neq 0$ and $y_0 \in Y$, there is a bounded linear map T from X to Y such that $T(x_0) = y_0$.
   (c) Let $x_1, x_2 \in X$. Show that if $x_1 \neq x_2$, then there is $T \in B(X,Y)$ such that $T(x_1) \neq T(x_2)$.
   (d) Show that $B(X,Y) \neq \{0\}$.
2. Let X be a normed space. Show that for any element $x_0 \in X$, we have $\|x_0\| = \sup\{|f(x_0)| : f \in X^*; \|f\| \leq 1\}$.

$* * *$ **End** $* * *$