HOMEWORK 1

NAME:

Problem 1: Integrate the function $f(x,y) = \sqrt{x}/y^2$ over the region $[0,4] \times [1,2]$.

Problem 2: Find the volume below the function $x/(1+xy)$ over $[0,1] \times [0,2]$.

Problem 3: Evaluate the integral

$$\int_0^1 \int_0^3 xe^{xy}\,dx\,dy.$$

Problem 4: Compute the following double integral.

$$\int_0^{\sqrt{\pi/3}} \int_0^1 xy\sin(xy^2)dxdy$$

Problem 5: Suppose that $f : [a,b] \to \mathbb{R}$ is an integrable function and

$$\int_a^b f = \lim_{\|P\| \to 0} L(P,f) = L.$$

Recall that this means that for every $\varepsilon > 0$, there exists $\delta > 0$ such that for every partition P of $[a,b]$, we have $|L(P,f) - L| < \varepsilon$. Show that if $g : [a,b] \to \mathbb{R}$ is another function such that $f = g$ except possibly at some fixed $x_0 \in [a,b]$, then g is also integrable and

$$\int_a^b g = \lim_{\|P\| \to 0} L(P,g) = L$$

as well. (Hint: Consider the cases $x_0 \in P$ and $x_0 \notin P$.)