THE CHINESE UNIVERSITY OF HONG KONG Department of Mathematics MATH4240 - Stochastic Processes - 2024/25 Term 2

Homework 8 Due date: April 23 Wednesday (11:59pm), 2025

Please submit online via Blackboard your answers to all SEVEN questions below including two added questions. The late submission will not be accepted. Reference solutions will be provided after grading.

Exercises (Chapter 3, Page 107): 16, 17, 18, 19, 21,

and

- Q1. A telephone booth has 1 telephone and 2 waiting spaces. Suppose people come in as a Poisson process with rate 2 per minute. Each one use the phone for 1 minute in average, and the usage time is an exponential random variable. Let X(t) denote the number of people in the booth at time t.
 - (a) Find the rate matrix for the process.
 - (b) In the long term, what is the probability that there are 2 persons in the booth?
- Q2. Suppose that the arrival rate at a checkout counter is 2 customers per minute. A single clerk is working at the counter and the service time is an exponential random variable with mean time 1/2 minute. However, if there are 3 customers or more, then someone will come to help and the service time reduces to a mean of 1/3 minute.
 - (a) Set up the queuing model in an infinite matrix.
 - (b) What is the stationary distribution of the queue?
 - (c) In the long term, what is probability that there are 4 customers waiting (including the one being served)?