Assignment 6

1.

(a) Prove that $|x|^\alpha$ is convex on $\mathbb{R}$ for $\alpha > 1$.

(b) Prove that the Quadratic-Over-Linear function $f(x,y) = \frac{x^2}{y}$ is convex on $\mathbb{R} \times (0, +\infty)$.

(c) Prove that the quadratic function

$$f(x) = \frac{1}{2}x^T Px + q^T x + r$$

is convex if P is symmetric and positive semi-definite.

2. Calculate the subdifferential of the following functions:

(a) $I(x) = \begin{cases} 0, & \text{if } x \in X \\ +\infty, & \text{if } x \notin X \end{cases}$, where $X \subseteq \mathbb{R}^N$ is a convex set.

(b) $f(x) = \begin{cases} 0 & \text{if } x \in [-1, +1] \\ |x| - 1 & \text{if } x \in [-2, -1) \cup (1, 2] \\ +\infty & \text{if } x \in (-\infty, -2) \cup (2, +\infty) \end{cases}$

3. It is known that for a convex function f and a closed bounded set K contained in $\mathrm{ri}(\mathrm{dom}\, f)$, f is Lipschitz continuous on K. Using counter examples to explain why either of the three following conditions cannot be removed to guarantee the Lipschitz continuity of f on K: (1) closedness, (2) boundedness, and (3) $K \subseteq \mathrm{ri}(\mathrm{dom}\, f)$ are essential.