Assignment 5

1. Prove that if $(f_i)_{i\in I}$ is a family of convex functions $\mathcal{X} \to [-\infty, +\infty]$, with I any set of indices, then $\sup_{i\in I} f_i$ is convex.

2. Prove the following propositions.

(a) Let $f_1, f_2, \ldots, f_k$ be convex functions and $w_1, w_2, \ldots, w_k$ be nonnegative real numbers. Prove that $f(x) = \sum_{i=1}^{k} w_i f_i(x)$ is a convex function.

(b) Use the definition of convexity to show that if $f_1, f_2 : \mathbb{R}^N \to \mathbb{R}$ are convex, then $\max(f_1, f_2)$ is convex.

3. Prove the following implication: strong convexity $\Rightarrow$ strict convexity $\Rightarrow$ convexity. Determine whether the converse of either implication is true and explain your reasons.
Remark. A function $f : \mathbb{R}^N \to \mathbb{R}$ is said to be strongly convex if there exists a constant $\rho > 0$ such that $f - \frac{\rho}{2}\|\cdot\|$ is convex.