Assignment 3

1. Prove that the following sets are convex:

(1) Intersections of convex sets. Let $X_i, i \in I$, be nonempty convex subsets. Show that $\bigcap_{i \in I} X_i$ is convex.

(2) Weighted sums of convex sets. Let $X_1, \ldots, X_k \subseteq \mathbb{R}^n$ be nonempty convex subsets and $\lambda_1, \ldots, \lambda_k$ be reals. Show that the set

$$\lambda_1 X_1 + \ldots + \lambda_k X_k \equiv \{x = \lambda_1 x_1 + \ldots + \lambda_k x_k : x_i \in X_i, 1 \leq i \leq k\}$$

is convex.

2. Show that the interior and closure of a convex set is also convex.

3. Show that the image and inverse image of a convex set under a linear transformation is also a convex set.