Assignment 7

1. Consider the problem

$$\min_{Ax=b} \frac{1}{2} \|x\|^2$$

where A is an $m \times n$ matrix and b is a vector in \mathbb{R}^m .

- (a) Calculate the dual problem (D).
- (b) Under what condition does the dual problem have a unique solution?
- (c) Calculate, in this case, this solution as a function of A and b.
- 2. Consider the support vector machine (SVM) problem:

$$\min_{w \in \mathbb{R}^N, b \in \mathbb{R}} \frac{\|w\|^2}{2}, \quad \text{subject to } v^{(i)}(w^T u^{(i)} + b) \ge 1, i = 1, \dots, m,$$

where $u^{(i)} \in \mathbb{R}^N, v^{(i)} \in \mathbb{R}$ are given. Derive the dual SVM problem and explain how to solve the primal SVM problem with the optimal solution to the dual SVM problem.

3. Solve the problem using duality

$$\min \quad \frac{1}{2} \left[(x-2)^2 + y^2 + z^2 \right]$$

subject to
$$x^2 + y^2 \le 1$$
$$y + z \le 0$$