

(by composing the maps here with translations & rotations.)



 $\underline{Eg4}$: $Z \vdash = ligZ$ branch defined by deleting $\{X < 0\}$ (i.e. $-\pi < argZ < \tau$)

maps It confirmally to strip of w=utiv: DKU<TT'S.



By the choice of the branch, for $z = re^{i\theta}$, $o < \theta < \overline{i}$ $\log z = \log r + i\theta$ \therefore $u = \log r \in \mathbb{R}$ and $v = \theta \in (0, \overline{i})$ The inverse is $w \mapsto e^{w}$.

Eq.5 Same $Z \rightarrow \log Z$ maps $D^{\dagger} = \{ Z = X + i y : |Z| < 1, y > 0 \}$ confamally to thalf strip $j = u + i v = u < 0, 0 < v < \pi \},$ since $U = \log r < 0$. $\int_{1}^{1} \int_{1}^{1} \int_{1}^{1}$







(mispruit in the textbook concerning the boundary bolianious, (confused with $z \mapsto \pm (z + \pm)$)





1,2 The Dirichlet Problem in a Strip

Divichlet Problem in the open set JZ consists of solving

$$\int_{U=f \text{ on }\partial\Omega} \Delta U = f \text{ on }\partial\Omega$$

where
$$\Delta = \frac{2^{2}}{2\times 2} + \frac{2^{2}}{3}$$
 is the Laplacian (operator)
 $f = given$ (cartinuous) function on 2π .

(i.e. Divichlet Problem = Boundary Value Problem for the Laplace equation) Known Fact : Solution to Divichlet Problem in the unit disk D. Recall : using polar conducates

Let f be a cartinuous function on $\partial D = S'$. Then f can be represented as a (periodic) function of θ $f(\theta)$, $0 \le \theta \le 2\pi$.

Then the unique solution to
$$\int \Delta u = 0 \quad \text{in } D$$
$$u = f \quad \text{on } \partial D = S^{1}$$
is given by
$$u(r, \theta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} P_{r}(\theta - \varphi) f(\varphi) \, d\varphi$$
where
$$P_{r}(\theta) = \frac{1 - r^{2}}{1 - 2r(\theta - \theta)^{2}}$$
(See textbook for reference)

In this section, we illustrate flow to use confused maps and
the solution of Dirichlest problem in the unit disc to solve
Dirichlest Problem in a neare general domain 52 in C.

$$\frac{\text{Lemma 1.3}}{\text{IF}} : U \Rightarrow V \text{ holo. } (U, V \text{ open in C})$$

$$If u: V \Rightarrow C is flammaic (ie. $\Delta u=0$),
then $u \circ F: U \Rightarrow C$ is flammaic.
Pf Easy exercise using Chain rule and Cauchy-Riemann equation.
(Or observing that I hold. G a U s.t. Re(T = u.)
Dirichlet Problem in the strip $\Omega = \{x + iy = x \in \mathbb{R}, x < y < 1\}$
Then boundary $\exists \Omega$ of Ω consists of two components

$$L_0 = \{x + iy : U = 0 \le x : L_1 = \{x + iy = y = 1 \le 1\}$$
let $S_0 \ge L_0 \Rightarrow \mathbb{R}$ and $S_1 : L_1 \Rightarrow \mathbb{R}$ be cartinuous
functions (and represented as functions of x any)$$

We need to find u(x,y) such that $\begin{cases} \Delta u = 0 \quad \text{in } J2 \\ u(x,0) = f_0(x) \\ u(x,1) = f_1(x) \end{cases}$



<u>Step</u> : Find a conformal map $F: D \rightarrow \mathcal{I}$.



Easy calculation
$$\Rightarrow$$

 $G(z) = \frac{\dot{\iota} - e^{\pi z}}{\dot{\iota} + e^{\pi z}} : \Omega \Rightarrow D$
is the inverse $(G = F^{-1})$

Boundary behaviour : $\varphi: -\pi \to 0 \iff F(e^{i\varphi}): i+\omega \to i-\omega$ $\varphi: o \to \pi \iff F(e^{i\varphi}): -\omega \to +\infty (x-\alpha x_{\delta})$



Define
$$\widehat{f}: S' = \partial D \rightarrow R$$
 by
 $f_0(F(e^{i\varphi}))$, $0 < \varphi < \pi$
 $\widehat{f}(\varphi) = \begin{cases} f_1(F(e^{i\varphi}) - \widehat{\iota}), & -\pi < \varphi < 0 \\ 0, & \varphi = 0, \pm \pi \end{cases}$

Then by $\lim_{|X| \to \infty} f_0(X) = \lim_{|X| \to \infty} f_1(x) = 0$, \widehat{f} is continuous.

Using the solution to the Dirichlet problem in the unit die D, $\widehat{u}(w) = \frac{1}{2\pi} \int_{-\pi}^{\pi} P_r(\theta - \varphi) \widehat{f}(\varphi) d\varphi$

is a turnomic function in D with boundary value $\widetilde{u}|_{\partial \mathcal{D}} = \widehat{f} .$ Then Lemma 1.3 \Rightarrow $u = \widetilde{u} \cdot G : \mathcal{I} \to \mathbb{R}(C\mathbb{C})$ is the solution to the Dirichlet problem in the strip \mathcal{R} . More explicitly, we have

$$\mathcal{U}(x,y) = \frac{x\overline{u}\pi y}{z} \left(\int_{-\infty}^{\infty} \frac{f_0(x-t)}{(wh(\pi t) - (w)Ty)} dy + \int_{-\infty}^{\infty} \frac{f_1(x-t)}{(wh(\pi t) + (w)Ty)} dy \right)$$

$$(0 < Y < 1)$$

(Details mitted set Ex7 & discussion on page 216 in the Textbook)