THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH2230B/C - Complex Variables with Applications - 2024/25 Term 2

Homework 5
Due: Friday 21st February 2025, 23:59pm.

Note. For those of you who may not have the textbook, I have manually typed the questions below. However, make sure you always double check for typos if possible, and in which case please inform me by sending an email to bwang@math.cuhk.edu.hk; I would correct them immediately.

1. (P. 119, Q4) By definition, we have

$$\int_0^\pi e^{(1+i)x}\, dx = \int_0^\pi e^x \cos x\, dx + i \int_0^\pi e^x \sin x\, dx.$$

Evaluate the integral on the left-hand side and then taking real and imaginary parts of it to find the values of the two integrals on the right-hand side.

2. (P. 133, Q3) Evaluate

$$\int_C f(z)\, dz$$

where $f(z) = \pi \exp(\pi \overline{z})$ and C is the boundary of the square with vertices at the points $0, 1, 1+i$, and i, the orientation of C being in the counterclockwise direction.

3. (P. 133, Q4) Evaluate

$$\int_C f(z)\, dz$$

where

$$f(z) = \begin{cases} 1 & \text{when } y < 0, \\ 4y & \text{when } y > 0, \end{cases}$$

and C us the arc from $z = -1 - i$ to $z = 1 + i$ along the curve $y = x^3$.

4. (P. 133, Q6) Evaluate

$$\int_C f(z)\, dz$$

where $f(z)$ is the principal branch

$$z^i = \exp(i \operatorname{Log} z) \quad (|z| > 0, -\pi < \operatorname{Arg} z < \pi)$$

of the power function z^i, and C is the semicircle $z = e^{i\theta}$ $(0 \leq \theta \leq \pi)$.

5. (P. 138, Q2) Let C denote the straight line segment from $z = i$ to $z = 1$, and show that

$$\left| \int_C \frac{dz}{z^4} \right| \le 4\sqrt{2}$$

without evaluating the integral.

Suggestion: Observe that of all points on the line segment, the midpoint is the closest to the origin, that distance being $d = \sqrt{2}/2$.

6. (P. 138, Q5)

Let C_R be the circle $|z| = R$ $(R > 1)$, described in the counterclockwise direction. Show that

$$\left| \int_{C_R} \frac{\operatorname{Log} z}{z^2}\, dz \right| < 2\pi \left(\frac{\pi + \ln R}{R} \right),$$

and then use l'Hospital's rule to show that the value of this integral tends to zero as R tends to infinity.

Practice Problems (Do not turn in)

P.133: 9
P.139: 3, 8