THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH2230B/C - Complex Variables with Applications - 2024/25 Term 2

Homework 4
Due: Friday 14th February 2025, 23:59pm.

Note. For those of you who may not have the textbook, I have manually typed the questions below. However, make sure you always double check for typos if possible, and in which case please inform me by sending an email to bwang@math.cuhk.edu.hk; I would correct them immediately.

1. **(P. 103, Q1)** Show that
   (a) $(1+i)^i = \exp\left(-\frac{\pi}{4} + 2n\pi\right)\exp\left(i\frac{\ln 2}{2}\right)$ $(n = 0, \pm 1, \pm 2, \ldots)$;
   (b) $\frac{1}{i^{2i}} = \exp[(4n+1)\pi]$ $(n = 0, \pm 1, \pm 2, \ldots)$.

2. **(P. 103, Q6)** Show that if $z \neq 0$ and a is a real number, then $|z^a| = \exp(a \ln |z|) = |z|^a$, where the principal value of $|z|^a$ is to be taken.

3. **(P. 103, Q9)** Assuming that $f'(z)$ exists, state the formula for the derivative of $c^{f(z)}$.

4. **(P. 108, Q8)** Point out how it follows from expressions
$$|\sin z|^2 = \sin^2 x + \sinh^2 y,$$
$$|\cos z|^2 = \cos^2 x + \sinh^2 y,$$
that
$$(a)\ |\sin z| \geq |\sin x|; \quad (b)\ |\cos z| \geq |\cos x|.$$

5. **(P. 108, Q9)** With the aid of expressions
$$|\sin z|^2 = \sin^2 x + \sinh^2 y,$$
$$|\cos z|^2 = \cos^2 x + \sinh^2 y,$$
show that
$$(a)\ |\sinh y| \leq |\sin z| \leq \cosh y; \quad (b)\ |\sinh y| \leq |\cos z| \leq \cosh y.$$

6. **(P. 114, Q3)** Solve the equation $\cos z = \sqrt{2}$ for z.

Practice Problems (Do not turn in)

P.103:   7
P.108:   14, 16