What is the different between the correct statement in definition of Riemann integrable

$$\boxed{\begin{array}{l} \exists\, L \in \mathbb{R} \text{ such that} \\ \quad \forall\, \varepsilon > 0,\ \exists\, \delta_\varepsilon > 0 \text{ such that} \\ \quad \forall \text{ tagged partition } \dot{\mathcal{P}} \text{ of } [a,b] \text{ with } \|\dot{\mathcal{P}}\| < \delta_\varepsilon, \\ \qquad\qquad |S(f;\dot{\mathcal{P}}) - L| < \varepsilon. \end{array}}$$

and the incorrect statement: ?

$$\boxed{\begin{array}{l} \forall\, \varepsilon > 0,\ \exists\, \delta_\varepsilon > 0 \text{ such that} \\ \forall \text{ tagged partition } \dot{\mathcal{P}} \text{ of } [a,b] \text{ with } \|\dot{\mathcal{P}}\| < \delta_\varepsilon, \\ \qquad\qquad |S(f;\dot{\mathcal{P}}) - L| < \varepsilon \text{ for some } L \in \mathbb{R} \end{array}}$$

(with this incorrect statement, 8 out of 10 points will be deducted in Question 6(a) in the midterm.)

Note that in a logical statement involving several quantifiers: "$\forall$" & "$\exists$", the order of "$\forall$" & "$\exists$" cannot be interchanged!

In the incorrect statement above, one can only required to find one $L \in \mathbb{R}$ s.t.

$$|S(f;\mathcal{P}) - L| < \varepsilon \text{ is correct}$$

(after given $\varepsilon > 0$, $\mathcal{P}$ and so on).

But it is trivially satisfied, because $S(f;\mathcal{P})$ is a real number, and taking $L = S(f,\mathcal{P}) \in \mathbb{R}$, we have $|S(f;\mathcal{P}) - L| = 0 < \varepsilon$.

So every function f satisfies the incorrect statement, which is clearly a "big" mistake.