Math 2058, HW 1. Due: 24 Sep 2022, before 11:59 pm

(1) Using the Axioms to show that
  (a) for all $a \in \mathbb{R} \setminus \{0\}$, $1/(1/a) = a$.
  (b) If $a > b > 0$, then $0 < a^{-1} < b^{-1}$.

(2) If A is a non-empty subset of $\mathbb{R}$ such that A is bounded from above. If we denote $-A = \{-a : a \in A\}$, show that $\inf(-A)$ exists and equals to $-\sup A$.

(3) Show that if A, B are bounded subsets of $\mathbb{R}$. Show that

$$\sup(A + B) = \sup A + \sup B$$

where $A + B = \{a + b : a \in A, \;\; b \in B\}$. Do we have

$$\sup A \cdot \sup B = \sup(A \cdot B)$$

where $A \cdot B = \{ab : a \in A, b \in B\}$? Justify your answer.

(4) Let X be a non-empty set and $f, g : X \to \mathbb{R}$ be two real valued function with bounded ranges. Show that

$$\sup\{f(x) + g(x) : x \in X\} \le \sup\{f(x) : x \in X\} + \sup\{g(x) : x \in X\}.$$

Give an example showing that the inequality can be a strict inequality.

(5) Show by using completeness that there is $x \in \mathbb{R}$ so that $x > 0$ and $x^3 + x = 5$. Show that such x is unique.