Math 2050, HW 2. Due: 10 Oct 2024, before 23:59

(1) Use the ε-N terminology, show the followings:

  (a) $\lim_{n\to+\infty} \dfrac{n^3+2n+1}{n^3-2} = 0.$

  (b) $\lim_{n\to+\infty} n^2 3^{-n} = 0.$

(2) Suppose (x_n) is a sequence of real number such that $x_n \to x$ for some $x \in \mathbb{R}$.

  (a) If $x_n \in [a,b]$ for some a, b, show that $x \in [a,b]$.

  (b) If $x \in (a,b)$, show that there exists N such that $x_n \in (a,b)$ for all $n > N$.

(3) Show that if $z_n = (a^n + b^n)^{1/n}$ for some distinct $a, b > 0$, then $z_n \to \max\{a,b\}$.

(4) Suppose (x_n) is a sequence of positive real number such that $x_n^{1/n} \to L$ for some $L \in [0,1)$. Show that $x_n \to 0$ as $n \to +\infty$. What if $L = 1$, what can you conclude? Justify your answer by either proving this or giving a counter-example.