Math 2050, HW 1. Due: 24 Sep 2022, before 23:59

(1) Using the Axioms to show that for all $a, b \in \mathbb{R}$,

$$(-a)^2 = a^2 \quad \text{and} \quad (a + (-b))^2 = a^2 + (-2ab) + b^2.$$

(2) If $x > 0$, show that there exists $n \in \mathbb{N}$ such that $3^{-n} < x$.

(3) Show that if A, B are bounded subsets of $\mathbb{R}$. Show that

$$\sup(A + B) = \sup A + \sup B$$

where $A + B = \{a + b : a \in A, \ \ b \in B\}$. Do we have

$$\sup A \cdot \sup B = \sup(A \cdot B)$$

where $A \cdot B = \{ab : a \in A, b \in B\}$? Justify your answer.

(4) Let X be a non-empty set and $f, g : X \to \mathbb{R}$ be two real valued function with bounded ranges. Show that

$$\sup\{f(x) + g(x) : x \in X\} \leq \sup\{f(x) : x \in X\} + \sup\{g(x) : x \in X\}.$$

Give an example showing that the inequality can be a strict inequality.

(5) Show by using completeness that there is an unique $x \in \mathbb{R}$ so that $x > 0$ and $x^3 = 4$.