

Math 2050, HW 4 (due: 15 Nov, before 23:59)

- (1) By using ε - δ terminology, Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by

$$f(x) = \frac{x^2}{x^2 + 2x + 2}$$

is continuous.

- (2) Let $f : (0, +\infty) \rightarrow \mathbb{R}$ be a function given by

$$f(x) = \begin{cases} \frac{1}{m^2}, & \text{if } x = \frac{m}{n}, \gcd(m, n) = 1; \\ 0 & \text{otherwise.} \end{cases}$$

Determine the set of c where f is continuous at c .

- (3) Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$ is a function such that for some $K > 0$, we have

$$|f(x) - f(y)| \leq K|x - y|$$

for all $x, y \in \mathbb{R}$. Show that f is continuous. Is the converse true? Justify your answer.

- (4) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x) = 0$ for all x in form of $m2^{-n}$ for some $m \in \mathbb{Z}$ and $n \in \mathbb{N}$. Show that f is identically zero if f is continuous.
- (5) Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$ be two continuous function. Show that $h(x) = \max\{f(x), g(x)\}$ is also a continuous function.